

# Modelling of micro-machining of Ti-6Al-4V: strain gradient interpretation

Rahul Yadav, Gautam Kumar\*, Nilanjan Das Chakladar, Soumitra Paul

Indian Institute of Technology Kharagpur, Kharagpur, India

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## ABSTRACT

### KEYWORDS

Micro-Cutting,  
Residual Stress,  
Chip Morphology,  
Ti-6Al-4V.

*Advances in computer methods over the last two decades have accelerated research in engineering sectors because of high computing power. Micro-machining is a manufacturing domain that is widely utilized for producing miniature components where predictability is a concern. The current work emphasized developing and executing a user-defined constitutive flow and friction models to simulate the physical phenomenon of chip morphology, residual stresses, and cutting forces during orthogonal machining at the micro-scale. The proposed model integrates strain gradient and dynamic recrystallization effect using a user hardening subroutine written in Fortran for machining of Ti-6Al-4V in micron scale. Furthermore, a user defined friction subroutine was implemented at the tool-chip interaction. A comparison is made between the modelling results and experiments in terms of specific cutting energy (SCE) and residual stresses.*

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## 1. Introduction

The manufacturing of miniaturized components in the engineering and biomedical sectors became popular due to the advent of lithography and non-lithography-based techniques such as electro-discharge machining, laser micro-manufacturing, LIGA, electroplating, ion beam, and ultrasonic machining (Chae et al., 2006). Lithography-based techniques can produce exact geometrical features with limited tolerance for Si material. However, these techniques are not cost-effective, and their processing time is extremely high. The technique of micro-mechanical machining overcomes the challenges of such processes.

Since the depth of cut and feed during the micro-cutting process are substantially more prominent than the microstructure arrangement of grains of the workpiece, the material is considered as homogeneous and isotropic. However, the level of strain gradient and the local strain affect the flow stress (Wang et al., 2008). The finite element analysis (FEA) is a reliable numerical method for studying a complicated nonlinear system such as a material removal process. The accuracy of FE

modelling reckons on the selection of material and friction model. Özel and Karpaz (2007) illustrated the procedure to predict the Johnson-Cook constitutive parameters for different materials during the micro-cutting process. An enhanced strain softening model was introduced and the orthogonal turning process was studied to analyze chip morphology and the machining forces of Ti64 (Calamaz et al., 2010; Sima & Özel, 2010). Harzallah et al. (2017) introduced a new damage model, and postulated maximum shear criteria for better prediction of chip serration and shear band evolution during cutting of Ti64. Liu et al. (2021) modified the Johnson-cook constitutive model by adding the microstructure evolution, which considered the hardening and dynamic recovery during orthogonal machining of Ti64. Yadav et al. (2022a) proposed the models for plastic flow of material and friction by considering the dynamic recrystallization and stick-slip oscillations at the contact of chip and tool while micro-cutting Ti-6Al-4V.

Most of the aforesaid studies reported the FE modelling of the micro-machining operation with the Johnson-Cook and strain-softening constitutive material model. However, strain gradient (nonlocal effect) significantly influenced the micro-cutting process. Further, an elementary Coulomb friction model was described to define

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\*Corresponding author E-mail: gautam.a2k15@gmail.com

the chip-tool interface. Therefore, in this proposed study, the micro-machining process is simulated by considering the dynamic recrystallization and strain gradient effect. An enhanced user-defined stick-slip friction model is used at the chip and tool interaction. The comprehensive investigation of chip morphology, machining forces, SCE and residual stresses are proposed and verified with experimental results from reported literature.

## 2. Finite Element Modeling Approach: Micro-Scale Cutting

A two-dimensional orthogonal model with plane strain assumption is proposed to investigate the micro-cutting mechanism with an enhanced constitutive flow and friction model of Ti-6Al-4V. The cutting tool is considered to be rigid with an orthogonal rake angle of +8° and a cutting-edge radius of 1.5 μm. The assembly view with the meshing of parts for orthogonal cutting is illustrated in Fig. 1. Micro-cutting process deformed the material severely with a high strain rate. Thus, a dynamic explicit solver is implemented to solve the coupled temperature-displacement nonlinear differential equations.

### 2.1. Constitutive model

The plastic deformation of the workpiece with the coupled effect of temperature, strain rate, and strain can be illustrated by Johnson-Cook (J-C) model (Yadav et al., 2021). The material model is represented as below:

$$\sigma_{J-C} = \{A + B\varepsilon_p^n\} \left\{1 + C \ln\left(\frac{\dot{\varepsilon}_p}{\dot{\varepsilon}_{ref}}\right)\right\} \left\{1 - \left(\frac{T - T_{amb}}{T_{melt} - T_{amb}}\right)^m\right\} \dots(1)$$

During machining Ti-6Al-4V, materials suffer higher strain with an elevated strain rate of order 10<sup>3</sup> to 10<sup>5</sup> s<sup>-1</sup> and reflect strain softening and shear band formation. The J-C model unable to capture the proper chip morphology and plastic strain at this elevated strain rate. Thus, a TANH model proposed by Sima and Özel (2010) combines thermal softening and strain softening in the J-C model. The obtained expressions are given below

$$\sigma_{TANH} = \left[ A + B \varepsilon_p^n \left\{ \frac{1}{\exp(\varepsilon_p^r)} \right\} \right] \left[ 1 + C \ln\left(\frac{\dot{\varepsilon}_p}{\dot{\varepsilon}_{ref}}\right) \right] \left[ 1 - \left(\frac{T - T_{amb}}{T_{melt} - T_{amb}}\right)^m \right] \left[ D + (1 - D) \left\{ \tanh\left(\frac{1}{(\varepsilon_p + p)^r}\right) \right\}^s \right] \dots(2)$$

Where,  $D = \left\{ 1 - \left(\frac{T}{T_{melt}}\right)^d \right\}$ ,  $p = \left(\frac{T}{T_{melt}}\right)^b$ , and a, b, d, r, s indicates the material constants.

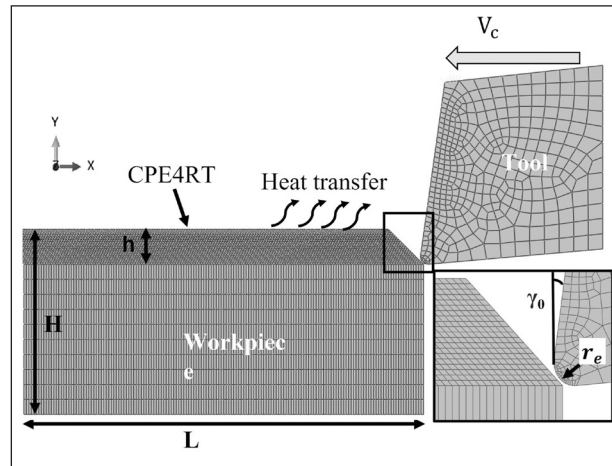


Fig. 1. Geometrical illustration of 2D model of orthogonal micro-machining.

The consequences of dynamic recrystallization were considered using the Johnson Mehl Avrami Kolmogorov (JMAK) model. The amount of recrystallized volume over the total element volume is mentioned below (Liu et al., 2021):

$$z_{dr} = 1 - \exp\left[-\beta \left(\frac{\varepsilon_p - \varepsilon_{critical}}{\varepsilon_{0.5}}\right)^k\right], \text{ if } \varepsilon_p > \varepsilon_{cr} \text{ and } z_{dr} = 0, \text{ if } \varepsilon_p \leq \varepsilon_{cr} \dots(3)$$

Where,  $\varepsilon_{cr} = \alpha_1 \alpha_2 \dot{\varepsilon}_p^{m_2} \exp\left(\frac{Qm_2}{RT}\right)$  and  $\varepsilon_{0.5} = \alpha_0 \dot{\varepsilon}_p^{m_1} \exp\left(\frac{Qm_1}{RT}\right)$

$\varepsilon_{critical}$  is the critical strain that symbolizes the growth of dynamic recrystallization amid the coupled thermo-mechanical analysis, and  $\varepsilon_{0.5}$  is the strain at  $z_{dr}=0.5$ . Q and R are energy of activation (kJ/mol) and universal gas constant (J/K mol), respectively. A hybridized flow model is stipulated using the J-C and TANH models, coupled with dynamic recrystallization. The hybridized model is implemented in ABAQUS as user subroutine VUHARD.

$$\sigma_{hybrid}^1 = (1 - z_{dr})\sigma_{J-C} + z_{dr} \sigma_{TANH} \dots(4)$$

In addition, the effect of inhomogeneous deformation is implemented by imposing a strain gradient in the flow model. The flow stress with strain gradient is discussed in equation 5 (Lai et al., 2008).

$$\sigma_{hybrid}^2 = \sigma_{hybrid}^1 \sqrt{1 + (\psi\eta)^k}, \text{ where, } \psi = \frac{2b(M\alpha G)^2}{(\sigma_{hybrid}^1)^2} \text{ and } \eta = \frac{4\varepsilon_{xy}}{a_1} \dots(5)$$

The factor of strain gradient ( $\eta$ ) relies upon the shear strain  $\varepsilon_{xy}$  and uncut chip thickness (uct)  $a_1$ . The characteristic length of deformation is defined by  $\psi$ , which relies on burgers vector  $b$ , Taylor's

**Table 1**

Material model and damage model parameters (Chen et al., 2011; Yadav et al., 2022b).

A (MPa) 1098	B (MPa) 1092	C 0.014	m 1092	n 0.47	$T_{amb}$ (°C) 25	$T_{melt}$ (°C) 1630	
$\dot{\epsilon}_{ref}$ (sec <sup>-1</sup> ) 0.01	a 2	b 5	d 0.5	r 2	s 0.05	Q (KJ/mol) 218	
$\beta$ 0.69	$\alpha_0, \alpha_1, \alpha_2$ 1.21e-05, 2, 0.5		$m_1, m_2$ 0.04, 0.006		k 2	R (J/K mol) 8.31	
$D_{c1}$ -0.09	$D_{c2}$ 0.25	$D_{c3}$ -0.5	$D_{c4}$ 0.014	$D_{c5}$ 3.87	b 0.3	M 3.06	$\alpha$ 0.4

factor  $M$ , the empirical constants  $\alpha$ , shear modulus  $G$ , and the flow model  $\sigma_{hybrid}^1$ . The workpiece (Ti-6Al-4V) material constants are mentioned in Table 1.

**2.2. Damage model**

The damage initiation criterion is captured by implementing J-C damage model (Zeng et al., 2017). The damage model parameters in the equation 6 for Ti-6Al-4V are given in Table 1. The fracture strain is given by  $\epsilon_p^f$ .  $\xi$  is the triaxiality of stress (the ratio of the hydrostatic pressure to the equivalent Von-Mises stresses). The aggregate model is defined as below:

$$\epsilon_p^f = [D_{c1} + D_{c2} \exp(D_{c3} \xi)] \left[ 1 + D_{c4} \ln \left( \frac{\dot{\epsilon}_p}{\dot{\epsilon}_{ref}} \right) \right] \left[ 1 + D_{c5} \frac{T - T_{amb}}{T_{melt} - T_{amb}} \right] \dots \dots \dots (6)$$

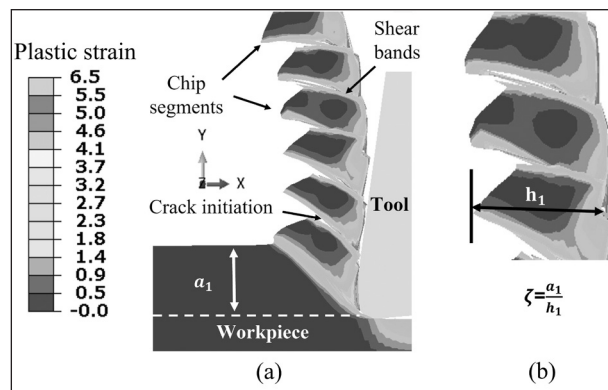
**2.3. Friction model**

During micro-machining, the frictional stick-slip phenomenon is observed at the interaction of the workpiece and the cutting edge of the tool. A FORTRAN code-based user subroutine (VFRIC) is employed to incorporate the stick-slip friction model into ABAQUS. The algorithm is illustrated below (Yadav et al., 2022b):

Sticking phenomenon ( $\tau < \mu P$ )  $\tau_{fr} = \mu_{st} \left( \frac{\gamma}{\gamma_{cr}} \right) \sigma_n$  .....(7)

Slipping phenomenon ( $\tau \geq \mu P$ )  
 $\tau_{fr} = \mu_{st} \sigma_n - (\mu_{st} - \mu_{kin}) \sigma_n \cos \left( \omega_f \left( \frac{\gamma}{\gamma_{cr}} \right) \right)$  .....(8)

Where  $\mu_{st}$  (0.33), and  $\mu_{kin}$  (0.2) are the static and kinetic coefficient of friction at the region of stick-



**Fig. 2.** Chip morphology analysis at  $V_c = 31.41$  m/min (a) chip segmentation (b) chip thickness.

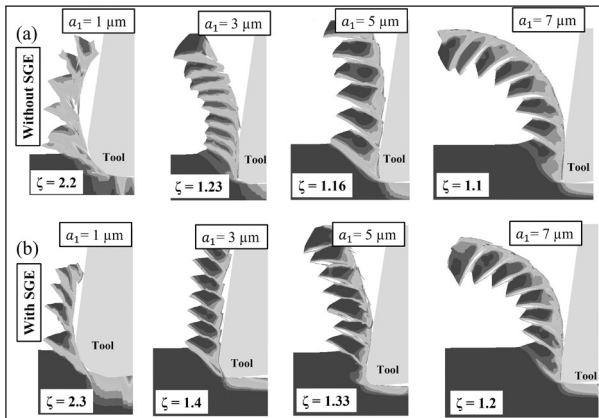
slip.  $\gamma_{cr}$ ,  $\dot{\gamma}_{cr}$  and  $\gamma$  symbolize the critical elastic slip, slip rate, and elastic slip.  $\tau_{fr}$ , and  $\sigma_n$  represent frictional stress and normal contact stress. The frequency of the chip segmentation by  $\omega_f$ .

**3. Results and Discussions**

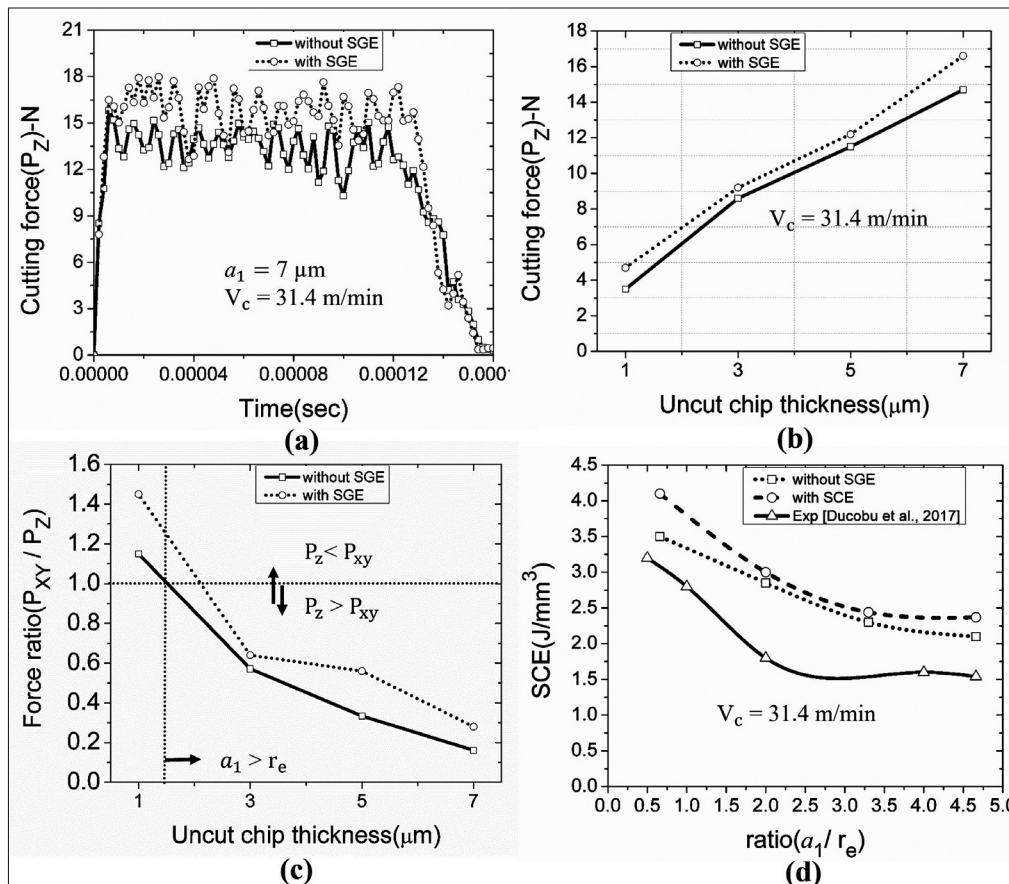
The estimation of the chip morphology, cutting force, SCE and residual stress at varying feed is discussed in this section. The machining of Ti-6Al-4V in micron scale results in formation of serrated chip, as depicted in Fig. 2. The crack initiation and the shear band are observed at the chip segments. The maximum plastic strain of 5.5 was localized along the shear band. The chip morphology with varying uct ( $a_1$ ) is depicted in Fig. 3 with and without consideration of the strain gradient effect. The average value of the coefficient of chip reduction ( $\zeta = h_1/a_1$ ) decreased with an increase in uct ( $a_1$ ) and was observed to be maximum for  $1 \mu m$  uct ( $a_1 < r_e$ ) due to the size effect (Sima & Ozel, 2010). The rake angle near the edge radius became negative, and the material underwent large compression (squeezing) while passing along

the edge radius. The  $\zeta$  value was higher for the strain gradient flow model due to high shear deformation at the micro-scale caused by the strain gradient effect. The variation of primary cutting force with time and uct ( $a_1$ ) is illustrated in Fig. 4(a) and Fig. 4(b), respectively. The primary cutting forces  $P_z$  and its orthogonal component  $P_{xy}$  affect the cutting power, tool life, and surface finishing,

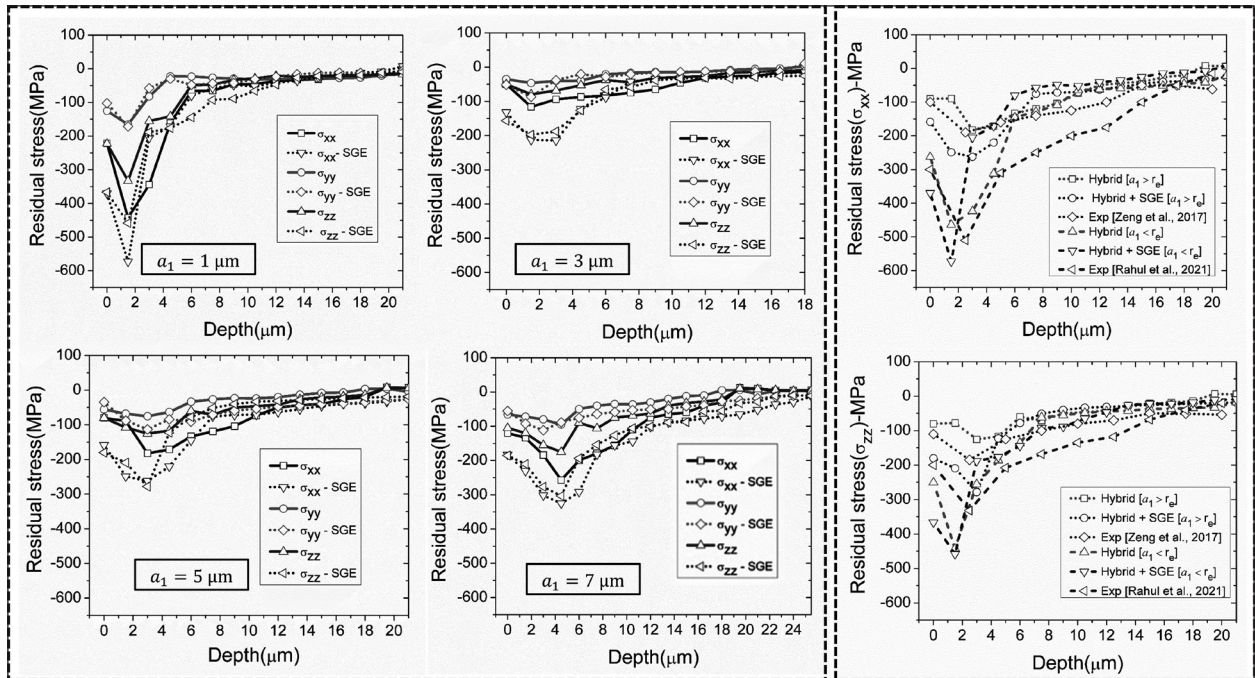
the variation of their ratios is shown in Fig. 4(c) for different uct ( $a_1$ ). The analysis reveals that the force ratio enhanced with a reduction in uct ( $a_1$ ) from  $7 \mu\text{m}$  to  $3 \mu\text{m}$ . The thrust force dominated the machining force due to the size effect when the uct ( $a_1 = 1 \mu\text{m}$ ) was less than the radius of the cutting edge ( $r_e = 1.5 \mu\text{m}$ ). The force ratio was higher with the strain-gradient effect due to higher flow stress during plastic deformation. Fig. 4(d) describes the SCE plot with varying  $a_1/r_e$  ratios. The SCE is inversely dependent on the material removal rate. As the uct ( $a_1$ ) reduces, the chip load decreases, significantly increasing the SCE at a low value of uct ( $a_1$ ). A similar trend was obtained by Ducobu et al. (2017) at the same ratio of  $a_1/r_e$  at the meso scale. The size effect reduces with an increase in undeformed chip thickness irrespective of the same scalar proportion of uct ( $a_1$ ) and edge radius, leading to the simulated trend being larger than the reported trend. The residual stress in micro-cutting is induced by non-uniform plastic deformation, temperature gradients, and phase change. In the proposed study, the effect of phase change on residual stress is ignored. The residual stresses in compression are usually observed



**Fig. 3.** Chip morphology at different uct  $a_1$  (a) without strain gradient effect (without SGE) (b) with the effect of strain gradient (with SGE).



**Fig. 4.** (a), (b) cutting force study with time and uncut chip thickness, respectively, (c) variation of force ratio analysis, (d) analysis on SCE ( $\text{J}/\text{mm}^3$ ) at  $V_c = 31.41 \text{ m}/\text{min}$ .



**Fig. 5.** Residual stresses variation with and without the effect of strain gradient at different uct (h) (a) 1  $\mu\text{m}$  (b) 3  $\mu\text{m}$  (c) 5  $\mu\text{m}$  (d) 7  $\mu\text{m}$ ; The validation of the proposed model with literature (e) along machining direction x (f) along transverse direction z.

at the micro-machined surface, with maximum magnitude at a certain depth due to the build-up of high-density of dislocations, and its magnitude further decreases with an increase in depth. The analysis of induced compressive residual stresses with uct ( $a_1$ ) in the machining direction is illustrated in Fig. 5 (a, b, c, d). The study revealed that the compressive residual stresses reduced with a reduction in uct ( $a_1$ ) from 7  $\mu\text{m}$  to 3  $\mu\text{m}$ . The size effect predominates at uct ( $a_1$ ) of 1  $\mu\text{m}$  ( $< r_e$ , 1.5  $\mu\text{m}$ ), resulting in a rise in the extent of compressive residual stress. The residual stress under the strain gradient effect was higher due to higher cutting forces (Fig.4) depicted by dotted lines.

The proposed FEM model was validated with the experimental work reported in the literature of Zeng et.al (2017) and Yadav and Mathew (2021) for  $\{(a_1 = 5 \mu\text{m}) > (r_e = 1.5 \mu\text{m})\}$  and  $\{(a_1 = 1 \mu\text{m}) < (r_e = 1.5 \mu\text{m})\}$  respectively in Fig 5 (e, f). The trend and the magnitude of compressive residual stresses from the FEM model in Fig 5 (e, f) satisfactorily matched with the proposed experimental work in the literature. The trend line comparison was made with the reported experimental data for  $a_1/r_e = 1.71$  and  $a_1/r_e = 0.88$ . The current work aimed to enhance and implement the constitutive flow material and friction model to capture the micro-machining phenomenon and machinability performance of Ti-6Al-4V.

#### 4. Conclusion

This proposed study develops a 2D finite element analysis for micro-machining with an enhanced flow and friction model incorporated through user-subroutine using Fortran. The cutting forces, chip morphology, SCE, and residual stress are analysed for Ti-6Al-4V. Some specific conclusions are mentioned below:

The chip serration and formation of the shear band for Ti-6Al-4V was simulated successfully through strain softening and strain gradient model. The implementation of strain gradient effect in the flow model captures the non-homogenous deformation during micro-machining, which results in increase of machining forces, SCE, and maximum residual stresses. The chip reduction coefficient was high ( $\zeta = 2.3$ ) for a lower uct ( $a_1$ ) of 1  $\mu\text{m}$  due to high shear deformation at low uct ( $a_1$ ). The cutting force ratio ( $P_{xy}/P_z$ ) increased rapidly from 0.65 to 1.5 with a reduction in uct ( $a_1$  value from 3  $\mu\text{m}$  to 1  $\mu\text{m}$ , and thrust force ( $P_{xy}$ ) dominated over cutting force ( $P_z$ ) caused by localized rake angle. The SCE showed the nonlinear response when uct is less than cutting edge ( $a_1 < r_e$ ) due to a drastic decrease in material removal rate. The surface residual stress (in magnitude) increased with an increase in uct ( $a_1$ ) value when  $a_1 > r_e$  due to high cutting forces. However, for  $a_1 \leq r_e$ , surface residual stress was enhanced due to the size effect.

## References

- Calamaz, M., Coupard, D., & Girod, F. (2010). Numerical simulation of titanium alloy dry machining with a strain softening constitutive law. *Machining Science and Technology*, 14(2), 244-257. <https://doi.org/10.1080/10910344.2010.500957>
- Chae, J., Park, S. S., & Freiheit, T. (2006). Investigation of micro-cutting operations. *International Journal of Machine Tools and Manufacture*, 46(3-4), 313-332. <https://doi.org/10.1016/j.ijmachtools.2005.05.015>
- Chen, G., Ren, C., Yang, X., Jin, X., & Guo, T. (2011). Finite element simulation of high-speed machining of titanium alloy (Ti-6Al-4V) based on ductile failure model. *International Journal of Advanced Manufacturing Technology*, 56(9-12), 1027-1038. <https://doi.org/10.1007/s00170-011-3233-6>
- Ducobu, F., Rivière-Lorphève, E., & Filippi, E. (2017). Experimental and numerical investigation of the uncut chip thickness reduction in Ti6Al4V orthogonal cutting. *Meccanica*, 52(7), 1577-1592. <https://doi.org/10.1007/s11012-016-0499-7>
- Harzallah, M., Pottier, T., Senatore, J., Mousseigne, M., Germain, G., & Landon, Y. (2017). Numerical and experimental investigations of Ti-6Al-4V chip generation and thermo-mechanical couplings in orthogonal cutting. *International Journal of Mechanical Sciences*, 134(October), 189-202. <https://doi.org/10.1016/j.ijmecsci.2017.10.017>
- Lai, X., Li, H., Li, C., Lin, Z., & Ni, J. (2008). Modelling and analysis of micro scale milling considering size effect, micro cutter edge radius and minimum chip thickness. *International Journal of Machine Tools and Manufacture*, 48(1), 1-14. <https://doi.org/10.1016/j.ijmachtools.2007.08.011>
- Liu, G., Zhang, D., & Yao, C. (2021). A modified constitutive model coupled with microstructure evolution incremental model for machining of titanium alloy Ti-6Al-4V. *Journal of Materials Processing Technology*, 297(June). <https://doi.org/10.1016/j.jmatprotec.2021.117262>
- Özel, T., & Karpuz, Y. (2007). Identification of constitutive material model parameters for high-strain rate metal cutting conditions using evolutionary computational algorithms. *Materials and Manufacturing Processes*, 22(5), 659-667. <https://doi.org/10.1080/10426910701323631>
- Sima, M., & Özel, T. (2010). Modified material constitutive models for serrated chip formation simulations and experimental validation in machining of titanium alloy Ti-6Al-4V. *International Journal of Machine Tools and Manufacture*, 50(11), 943-960. <https://doi.org/10.1016/j.ijmachtools.2010.08.004>
- Wang, J. S., Gong, Y. D., Abba, G., Chen, K., Shi, J. S., & Cai, G. Q. (2008). Surface generation analysis in micro end-milling considering the influences of grain. *Microsystem Technologies*, 14(7), 937-942. <https://doi.org/10.1007/s00542-007-0478-y>
- Yadav, R., K. V., & Mathew, J. (2021). Methodology for prediction of sub-surface residual stress in micro end milling of Ti-6Al-4V alloy. *Journal of Manufacturing Processes*, 62 (December 2020), 600-612. <https://doi.org/10.1016/j.jmapro.2020.12.031>
- Yadav, R., Chakladar, N. D., & Paul, S. (2022a). A dynamic recrystallization based constitutive flow model for micro-machining of Ti-6Al-4V. *Journal of Manufacturing Processes*, 77(March), 463-484. <https://doi.org/10.1016/j.jmapro.2022.03.040>
- Yadav, R., Chakladar, N. D., & Paul, S. (2022b). Micro-milling of Ti-6Al-4 V with controlled burr formation. *International Journal of Mechanical Sciences*, 231(July), 107582. <https://doi.org/10.1016/j.ijmecsci.2022.107582>
- Zeng, H. H., Yan, R., Peng, F. Y., Zhou, L., & Deng, B. (2017). An investigation of residual stresses in micro-end-milling considering sequential cuts effect. *International Journal of Advanced Manufacturing Technology*, 91(9-12), 3619-3634. <https://doi.org/10.1007/s00170-017-0088-5>



**Rahul Yadav** is a Ph.D research scholar working, under the supervision of Prof Soumitra Paul and Dr. Nilanjan Das Chakladar, in the field of modelling of micro-machining process at the Department of Mechanical

Engineering in the Indian Institute of Technology Kharagpur, India. His research comprises controlling burr generation in micro-milling and surface modification by ultrasonic-assisted cavitation peening. He has proposed and implemented the hybrid flow models and friction algorithm to simulate the physics of the micro-milling process and cavitation peening process. He has authored three journal articles, and 3 Conference presentations, and has a google scholar citation of 20+. (E-mail: rahul0223yadav@gmail.com)



**Gautam Kumar** is a Ph.D research scholar working, under the supervision of Dr. Nilanjan Das Chakladar and Prof. Soumitra Paul, in the field of hierarchical modelling of

grinding process at the Department of Mechanical Engineering in the Indian Institute of Technology Kharagpur, India. His research comprises of detailed understanding and implementation of constitutive and frictional behaviour of the workpiece to be ground in a finite element software and subsequently validate the results. He has recently been awarded the prestigious Prime Minister Research Fellowship to facilitate his Ph.D research work. He also received the SS Sahay award during his bachelor studies.



**Dr. Nilanjan Das Chakladar** is an Assistant Professor in the area of Manufacturing Science and Engineering in the Department of Mechanical Engineering at the Indian Institute of Technology Kharagpur,

India. His research interest includes composite manufacturing, composite process simulation and machining simulation. He has authored 12 journal articles, 15 Conference presentations, and has a google scholar citation of 450+. He is currently supervising 3 solo and 6 joint Ph.D scholars. (E-mail: ndaschakladar@mech.iitkgp.ac.in)



**Prof. Soumitra Paul** is a Professor of Manufacturing Science & Engineering in the Department of Mechanical Engineering at the Indian Institute of Technology Kharagpur, India.

His research interests include Surface finishing, micro-machining and design and fabrication of experimental test setups. He has authored 100+ journal articles and conference proceedings and has a google scholar citation of almost 4800+. He has completed supervision of 14 Ph.D scholars and currently supervising 3 scholars. (E-mail: spaul@mech.iitkgp.ac.in)