Thermal error modeling of machine tool spindle through an ensemble approach

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Support Vector Machine,
Linear Regression,
Neural Network.

ABSTRACT

Thermal error compensation of machine tool is cost-effective than other methods. Towards this, data-driven machine learning (ML) algorithms have been used to produce accurate prediction models. However, ML models have limitations, such as overfitting, requiring a large data etc. In present work, a hybrid model is proposed by exploiting the linear regression (LR), support vector machine (SVM), neural network (NN), and decision tree (DT) models. For this purpose, the optimum weights to each constituent model is identified by cosine similarity maximization. The developed models are validated against the experimental data. The prediction results with optimized weight are compared with equal weights and the root means square error (RMSE) for both methods are 1.8879 and 2.8978, respectively. The RMSE shows that the hybrid model produces good accuracy for both small and large data sets compared to individual models.

1. Introduction

Thermal error in machine tools contributes to 40%-70% of all machining errors. The spindle bearing, motor, machining zone, and ball screws are the main heat sources in a machine tool. Among them, spindle thermal error contributes to the maximum percentage of thermal error. Bearing type, friction torque, lubrication method, and spindle motor contribute to the total heat generation in a high-speed spindle. The heat flows through conduction, convection, and radiation, causing the thermal expansion to the spindle bearings, shaft and causing spindle thermal error during machining. Thermal error models are developed by establishing a functional relationship between temperature or heat generation and thermal error. Different data-driven models, such as linear regression (LR) (Lin et al., 2020), neural network (NN) (Zhang et al., 2012), and support vector machines (SVM) (Li et al., 2021) have been used for thermal error modeling. However, these models have their respective limitations, e.g., LR work with linear systems, and NN models require large amounts of data and come with overfitting issues. Towards this, different hybrid models (Lin & Fu, 2010) have been explored to address these issues by combining different data-driven models and extracting advantages from individual models. However, integration methods of these models still need to be explored to increase their reliability. Previous works have considered that the weights for combining the participant models are equal (Lin & Fu, 2010). The present work aims to develop a new hybrid model by combining the multivariable LR, SVM, NN, and decision tree (DT) models with the cosine maximization method to increase the accuracy of prediction.

2. Methodology

The methodology followed in realizing the proposed hybrid model was presented in the following:

- Individual constituent models, i.e., LR, SVM, and NN, were developed to predict thermal error by considering the training dataset.
- The cosine maximization method was applied to the predicted data to determine respective weights and realize a hybrid model by multiplying the weights with respective constituent models.
- The developed models have been tested and validated with different data sets to evaluate

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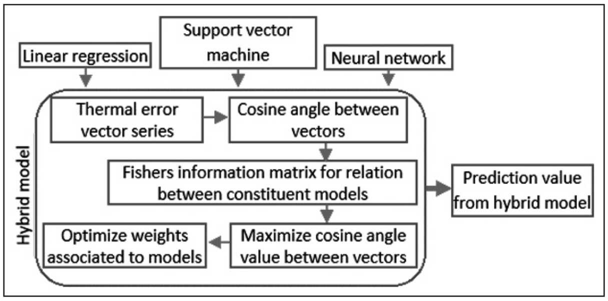

Fig. 1. Methodology for the development of hybrid models for the prediction of thermal error.

their prediction accuracy by evaluating the models' root mean square error (RMSE).

3. Development of Data-Driven Models for the Prediction of Spindle Drift/Thermal Error

The linear regression, support vector machines, neural networks and decision trees are primarily used to predict the thermal error of machine tools. Therefore, these three models have been used to develop the hybrid model in this work.

3.1. Linear regression

The linear regression establishes a linear relation between independent and dependent variables. For independent variables x_1, x_2..,x_n, and dependent variable *y* and the general form of the linear model is:

$$y = \alpha_0 + \alpha_1 x_1 + \alpha_2 x_2 + \cdots \ldots + \alpha_n x_n + \varepsilon \quad ..(1)$$

$$\varepsilon = \sum_{i=1} (y_i - \hat{y}_i)^2 \quad ....(2)$$

α_0 is a constant. α_1, α_2..., α_n are the corresponding coefficient of x_1, x_2..,x_n. Eq. 2 is used to calculate the sum of the least square error for all values of *y.*

3.2. Support vector machine

Support vector machine models usually predict with good accuracy and model in nonlinear systems with small sample data. A linear SVM was used in the present work to predict thermal error. SVM finds the optimal plane to meet the special requirements in the high dimensional decision space. The unique and global solution estimated from the SVM model can effectively prevent the NN from falling into local extreme values.

3.3. Neural network

The neural networks are generally used to model highly nonlinear systems. With proper weights and hidden layers, it can predict highly nonlinear behavior. A Levenberg-Marquardt-based neural network algorithm has been used in the present study. The algorithm is designed to approach second-order training speed without having to compute the Hessian matrix. As number of hidden layers would increase the computational time it has been taken as two and it was sufficient for required accuracy.

3.4. Decision tree

Decision tree-based machine learning algorithms are used in both classification and regression problems. Fine tree was used in the present work. However, it is primarily used in classification problems. It is a tree-structured classifier where internal nodes represent the features of a dataset, branches represent the decision rules, and each leaf node represents the outcome.

4. Hybrid Modelling Through Cosine Maximization

The present work proposes a cosine maximization method to combine different data-based models for thermal error modelling. Cosine similarity was evaluated between predicted values and actual values of thermal error. After that, the cosine similarities were maximized through the MATLAB®-based *'fmincon'* optimization method to develop a more accurate hybrid model by evaluating optimized weights for each constituent model.

$[x] = [x_1\ x_2\ x_3 \ldots\ldots. x_k]^T$ - Actual thermal error

$[\hat{x}] = [\hat{x}_1\ \hat{x}_2\ \hat{x}_3 \ldots\ldots. \hat{x}_k]^T$ - Predicted thermal model with the hybrid model

$[x_i] = [x_{i1}\ x_{i2}\ x_{i3} \ldots\ldots. x_{ik}]^T$ - Predicted thermal error with individual models

$$\hat{x}_t = \sum_{i=1}^{p} w_i\, x_{it} \quad ....(3)$$

where t = 1,2,3 ...k; w_i = Constituent weights for each model, subjected to

(a) $0 \leq w_i \leq 1$

(b) $\sum_{i=1}^{p} w_i = 1$

4.1. Cosine similarity

Cosine similarity between two vectors α_i and α_j in an *n*-dimensional vector space V which is mapped from V×V to a range of [0,1]. Hence similarity measure between two vectors α_i and α_j SM (w_i, w_j) ϵ [0,1]. The following properties need to be followed according to the above definition (Kou & Lin, 2014):

$$\forall\, a_i \in V, SM(a_i, a_j) = 1 \qquad \text{.............(4)}$$

$$\forall\, a_i, a_j \in V, SM(a_i, a_j) = 0 \qquad \text{.............(5)}$$

$$\forall\, a_i, a_j, a_k \in V, SM(a_i, a_j) \leq SM(a_j, a_k) \qquad \text{.............(6)}$$

Then cosine similarity measure between two vectors α_i and α_j as

$$cos(\theta) = \rho = \frac{\sum_{k=1}^{n} a_{ik} a_{jk}}{\sqrt{\sum_{k=1}^{n} a_{ik}{}^2} \sqrt{\sum_{k=1}^{n} a_{jk}{}^2}} \qquad \text{............(7)}$$

Similarly, cosine similarity vector measure of the actual thermal error value $[x]$ and predicted thermal error value from the hybrid model $[\hat{x}]$ is represented as

$$\rho = \frac{\sum_{k=1}^{n} x_t \hat{x}_t}{\sqrt{\sum_{k=1}^{n} x_t{}^2} \sqrt{\sum_{k=1}^{n} \hat{x}_t{}^2}}$$

$$\rho = \frac{\sum_{k=1}^{n} x_t (\sum_{i=1}^{p} w_i\, x_{it})}{\sqrt{\sum_{k=1}^{n} x_t{}^2} \sqrt{\sum_{k=1}^{n} (\sum_{i=1}^{p} w_i\, x_{it})^2}}$$

$$\rho = \frac{\sum_{i=1}^{p} w_i \sum_{k=1}^{n} x_t x_{it}}{\sqrt{\sum_{k=1}^{n} x_t{}^2} \sqrt{[W(IM)W^T]}} \qquad \text{..........(8)}$$

Information matrix = $IM = \sum_{t=1}^{k} x_{it} x_{jt}$

Weight matrix = W

The ρ has been maximized with the MATLAB® optimization tool *fmincon*, and optimum weights for the constituent models were evaluated.

5. Experimental Setup

The experimental setup is shown in Fig. 2. The experimental data has been extracted from the literature (Liu et al., 2017, 2020) WebPlotDigitizer 4.6® was used to extract the data points form literature. The extracted data points were fitted to cubic spline and then extrapolated by piecewise polynomial method according to required datapoints. The MATLAB® functions was used to perform the data extraction activities. Five temperature measurement points (T1, T2, T3, T4, T5) on a Leaderway V-450 CNC machine tool spindle, one measurement point for ambient temperature (T6), and displacement sensor for measurement of thermal deformation (E) in the z-axis of the spindle has been taken as input to train the models. It was shown that the thermal error was more sensitive to the T1 to T5 temperature. Three batches of data points have been extracted from the literature, i.e., K1 (Liu et al., 2020) K9, and K18 (Liu et al., 2017).

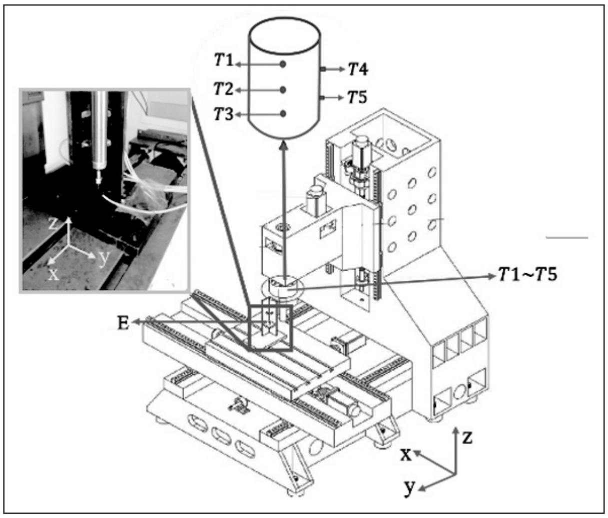

Fig. 2. Leaderway V-450 -temperature and error measurement points. (Liu et al., 2017, 2020)

Table 1
Operating conditions in different batches.

Parameters	K1	K9	K18
Spindle speed (rpm)	6000	6000	6000
Ambient temperature (°C)	7.5-11.7	9.81	21.6

The operating conditions are given in Table 1. The temperature and z-axis deformation data were taken for four hours at the interval of every 3 minutes (Case 1), as mentioned in the literature (Liu et al., 2017, 2020). The data was taken in 1 minute (Case 2), 20 seconds (Case 3), and 5 seconds (Case 3) intervals also to validate the models with data of different sampling rates.

6. Development of a Hybrid Model

Measured temperature and thermal error data for three batches were shown in Fig. 3. Training

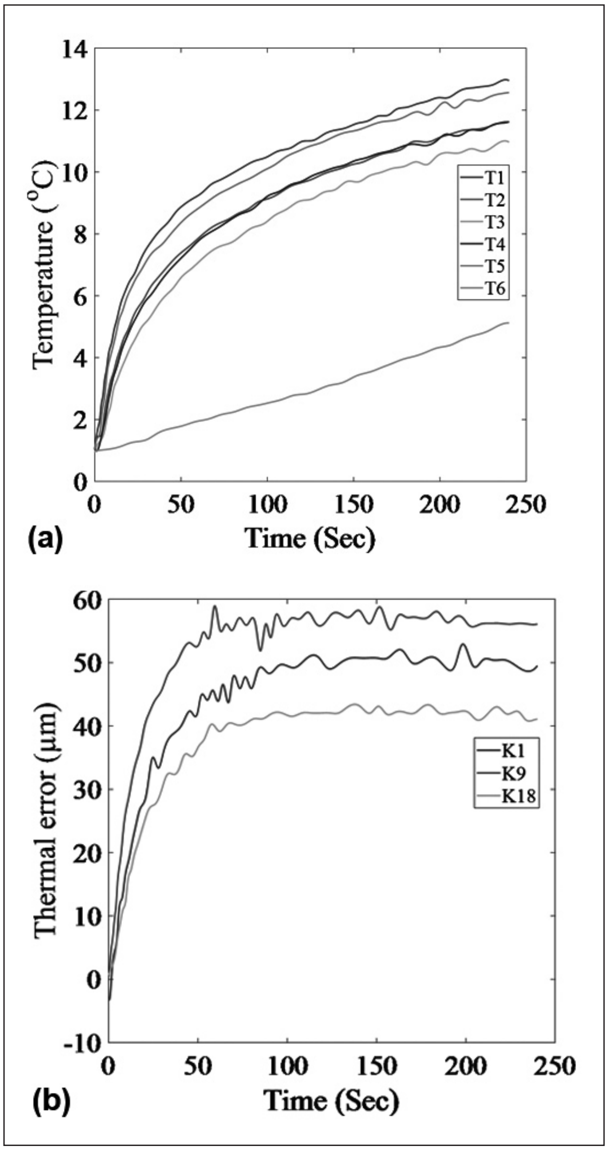

Fig. 3. Measured data for (a) temperature for the K1 batch and (b) thermal error for three batches.

data was formed with K9 and K18 batches. The data from two batches have been mixed and randomized. K1 batch data was used for test data. Details of training and testing data for the different sampling rates of data are presented in Table 2.

The data was used to develop LR, SVM, NN, and DT models. Hybrid model (E_HBD) was developed by multiplying predicted training data (E_LR, E_SVM, E_NN, E_DT) from trained models with respective weights (w1, w2, w3 and w4). The weights have been evaluated with the help of the cosine maximization method. The flow diagram for hybrid model development is given in Fig. 4.

Hereafter, the developed models are used to predict thermal error with test data (E1_LR, E1_SVM, E1_NN, E1_DT). The predicted thermal errors were multiplied with the weights (w1, w2, w3, and w4) evaluated at the training stage to predict thermal error (E1_HBD) through hybrid modelling. The hybrid model has also been developed with equal weights to see the improvement in thermal error prediction with optimized weights. The weight has been taken as 0.25 as the number of participant models is four.

Table 2
Training and prediction data.

Data type	Batch	Number of data points			
		Case 1	Case 2	Case 3	Case 4
Training	K9, K18	160	480	1440	5760
Testing	K1	80	240	720	2880

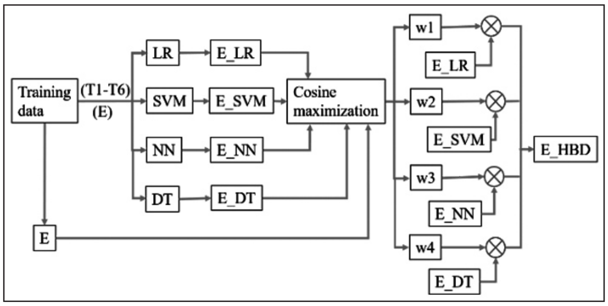

Fig. 4. Development of the hybrid model.

7. Results and Discussion

Predicted thermal errors for test data from different models for Case1 with optimized and equal weights is shown in Fig. 5 (a) and (b), respectively. The constituent models were seen to have lower accuracy for thermal error prediction. The models were combined with both optimized weights and equal weights to develop hybrid models. It can be seen from the figures that optimized weights perform better than equal weights. At the initial phase, when the thermal error increases rapidly with time, both models were predicting equally. However, in the transition to thermal stability, where the trend was highly nonlinear optimized weights give better accuracy in predicting thermal error. As the initial stage was almost linear, both cases can see good predictions.

Weight distribution among the constituent models is shown in Table 3. LR and SVM were assigned lower weights since they have lower accuracy than other constituent models in training phase. As LR and SVM has much lower weights, the model also considered combining only NN and DT. However, when LR and SVM model was not considered in present model as the biasness of the hybrid model is getting increased towards one of the remaining constituent models. It results in overfitting and consequently the testing accuracy of the model is being compromised. In order to avoid this, all four models have been included in the model.

Table 3
Weight distribution among the constituent models.

	LR	SVM	NN	DT
Optimum weight	0.0293	0.0255	0.7178	0.2274

The RMSE values for testing data was compared for both optimized and equal weights cases, and they are 1.8879 and 2.8978. Thus, optimized weights produce better accuracy in terms of RMSE values. The detailed values and plot have been shown in Table 4 and Fig. 6. It can be seen from Fig. 6. that the hybrid model produces less accurate results than NN in case of equal weight.

Table 4
Root mean square error for hybrid models: optimized and equal weights.

Testing data	LR	SVM	NN	DT	HBD
Optimized weights	7.3596	6.6382	2.6527	5.5413	1.8879
Equal weights	7.3596	6.6390	2.7503	5.5413	2.8978

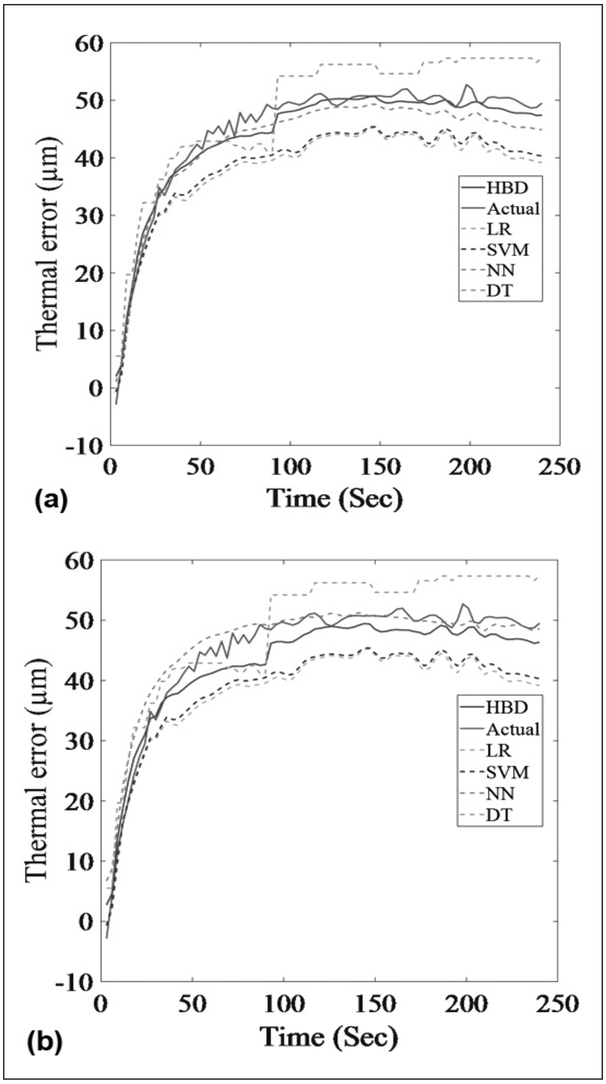

Fig. 5. Actual and predicted thermal errors for case 1 with (a) optimized and (b) equal weights.

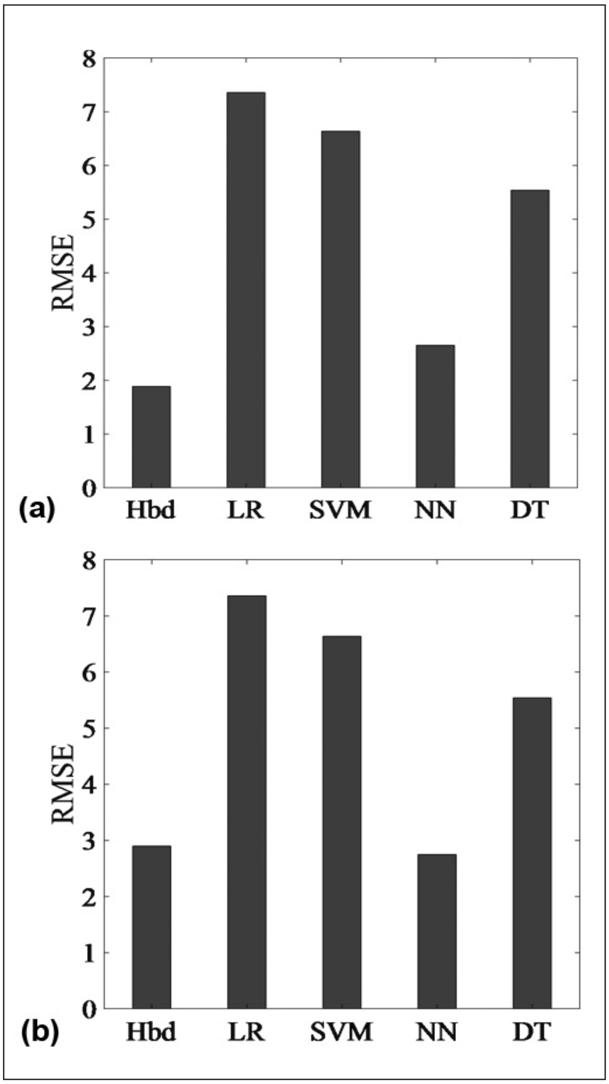

Fig. 6. Root mean square error for case 1 with (a) optimized weights and (b) equal weights.

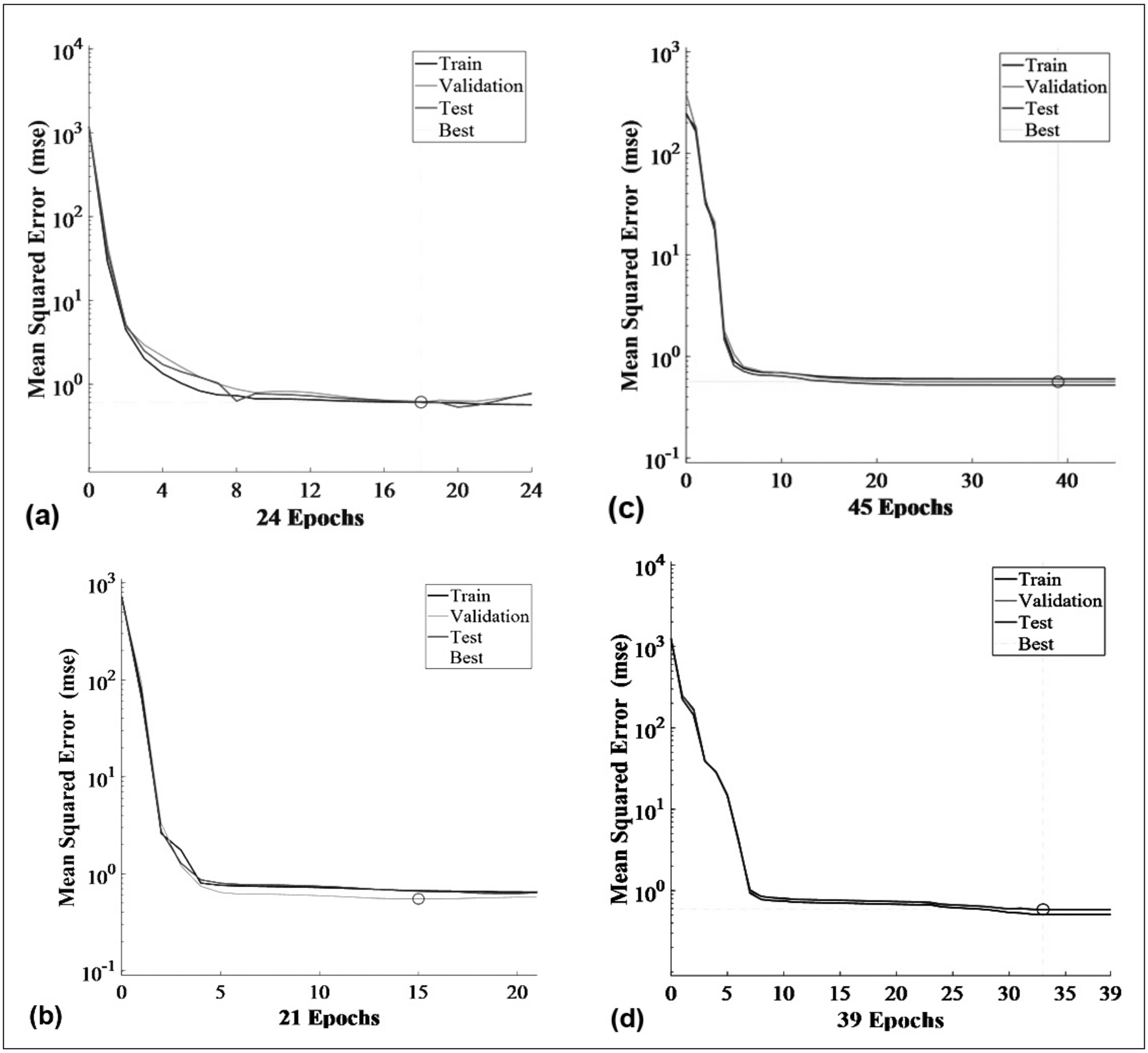

Fig. 7. MSE vs epoch in NN modelling for (a) Case 1, (b) Case 2, (c) Case 3, (d) Case 4.

Table 5
Best MSE for validation with respect to number of epochs.

	Case 1	Case 2	Case 3	Case 4
Epoch	18	15	39	33
MSE	0.61131	0.55173	0.56318	0.58557

The performance of the hybrid model approach has also been tested for comparatively large number of data points i.e. with Case 2, Case 3, and Case 4. The performance of neural network depends on number epoch and hidden layers. The mean square error for training and validation with epoch in NN modelling has been shown in Fig 7 (a), (b), (c) and (d) for all cases. Table 5 shows the optimal number of epochs for each case to achieve best model performance.

The predicted thermal errors for Case 2, Case 3, and Case 4 have been shown in Fig 8 (a), (b), and (c), respectively. In all three cases, a sharp change in slope can be seen in the DT model. As a decision tree is primarily used in classification problems, it can be assumed that it considers rapid temperature rise and thermal stable zone separately and predicts thermal error accordingly. The hybrid model also follows the DT model and gives maximum weightage as it has good training accuracy. The weight distribution among the constituent models has been shown for all three cases in Table 6. The LR and SVM were assigned the least weights as they have the least training accuracy.

The RMSE values for all three cases are shown in Table 7. The hybrid model shows good accuracy

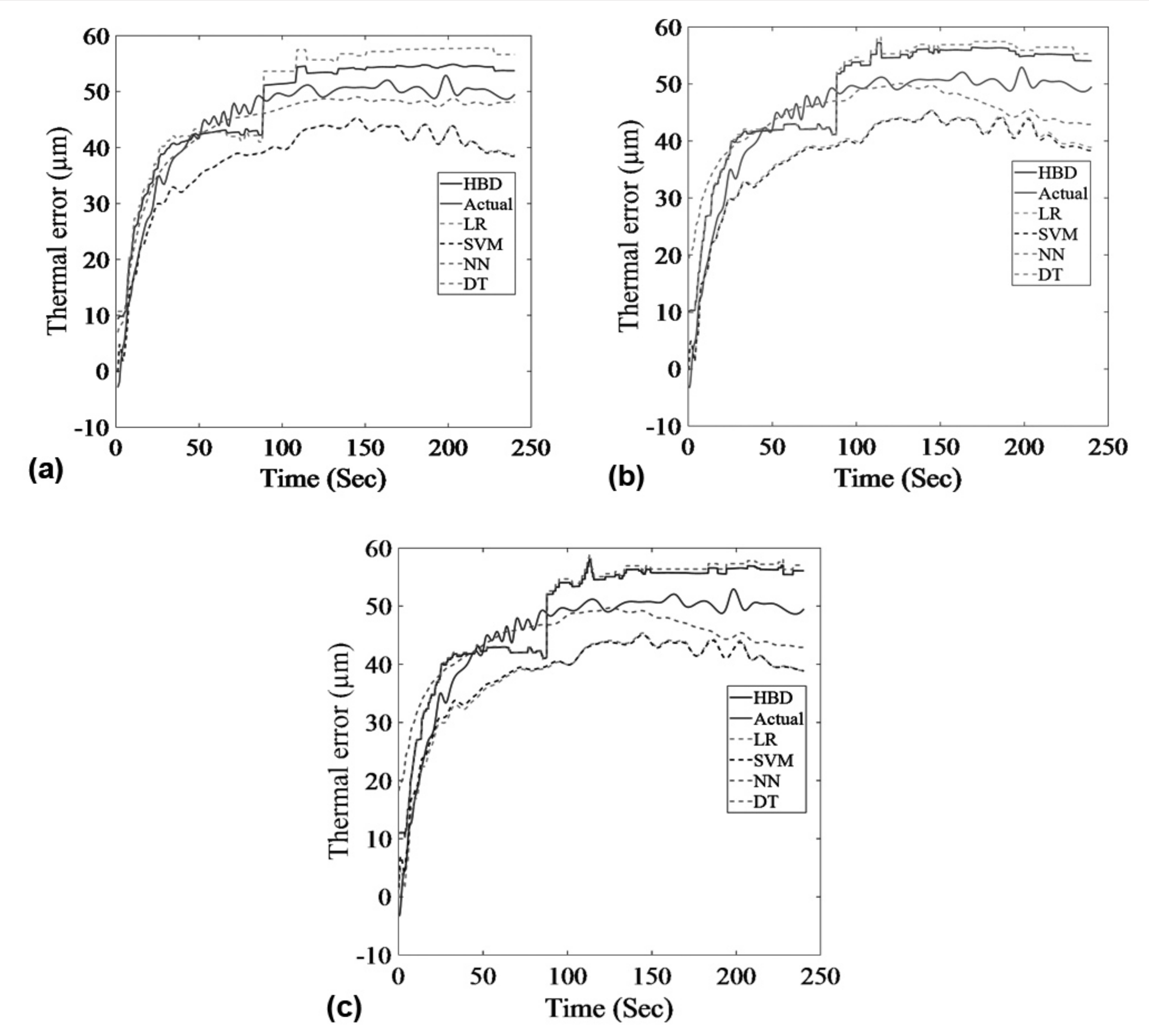

Fig. 8. Actual and predicted thermal errors for (a) Case 2, (b) Case 3, and (c) Case 4.

Table 6
Weight distribution among the constituent models for Case 2, Case 3 and Case 4.

Optimum weight	LR	SVM	NN	DT
Case 2	0.0182	0.0185	0.2617	0.7016
Case 3	0.0175	0.0177	0.0526	0.9122
Case 4	0.0147	0.0133	0.0323	0.9397

Table 7
Root mean square error for Case 2, Case 3, and Case 4.

RMSE	LR	SVM	NN	DT	HBD
Case 2	7.6447	7.7572	2.5633	6.1241	4.0765
Case 3	7.5062	7.8492	5.6330	5.7463	5.0865
Case 4	7.5054	7.4657	5.4821	5.8462	5.3563

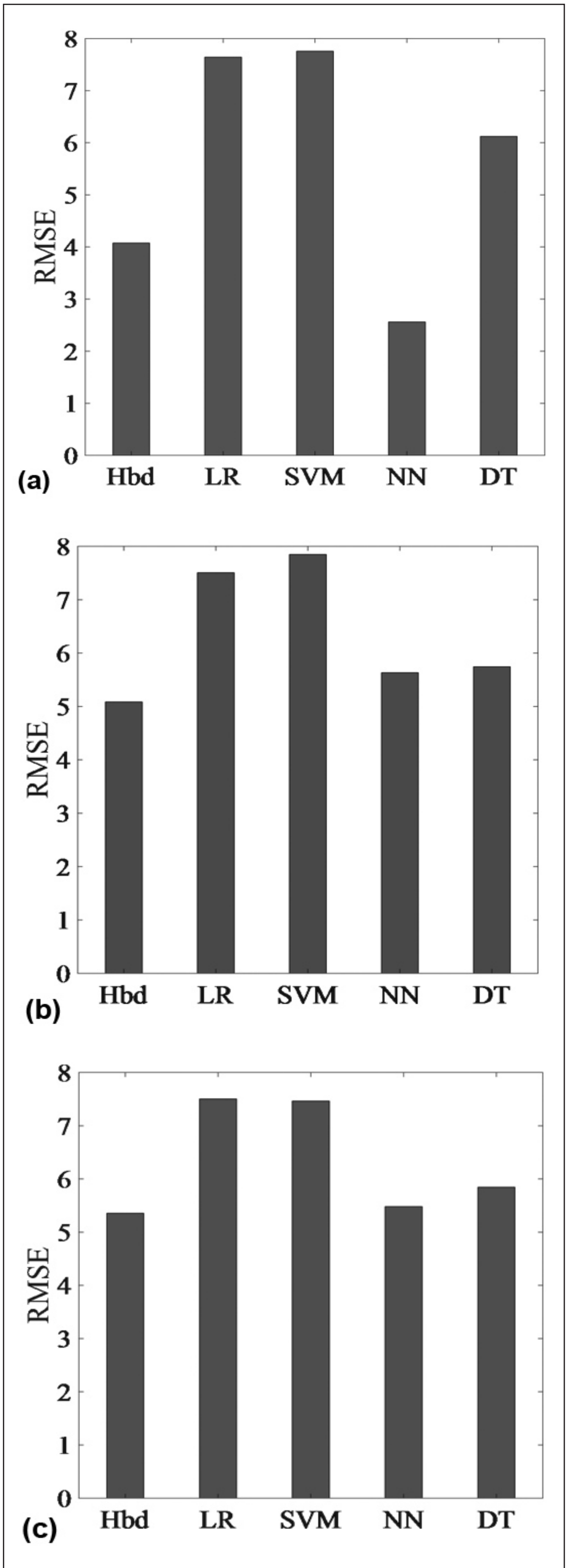

Fig. 9. Root mean square error for (a) Case 2 (b) Case 3 (c) Case 4.

compared to other models for all three cases. NN and hybrid show similar RMSE values in most cases. However, it can be seen from Fig. 9 that the hybrid model produces good accuracy in two distinct zones of rapid temperature rise and thermally stable zone; and the neural network shows good accuracy in transition. Thus, the hybrid model shows reliable prediction in most of the regions.

8. Conclusion

The present work aims to develop a hybrid thermal error model, combining LR, SVM, NN, and DT by considering the cosine similarity between actual thermal error and predicted error. The RMSE have been evaluated for the hybrid models and each constituent model to compare the accuracy. The hybrid model has shown good accuracy with low and high sampling rate data. The following highlights can be summarized from the present research:

- Equal weights have shown good accuracy with two constituent models in the literature. However, hybrid models with optimized weights have shown better prediction accuracy for three or more constituent models.
- Though NN and hybrid model shows similar RMSE values for larger number of data points, the latter shows better accuracy in rapid temperature rise at initial stage and thermally stable zone.
- The LR and SVM models have been assigned smaller weights as training accuracy for both the models was less than the other models.

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