Mathematical modeling and analysis of hoop stresses in hydro-forming deep drawing of n-sided polygonal AA1100, galvanized commercial steel cups

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ABSTRACT

KEYWORDS

Hydro-Forming Deep Drawing,
Hoop Stress,
n-Sided Polygonal Cup.

The main objective of this paper presents the analytical evaluation and mathematical modeling of hoop stresses of metallic cups of materials such as AA1100 and galvanized commercial steel in hydro forming deep drawing of n-sided polygonal cup. It is very important to find the magnitude of these stresses produced within the flange area. Two types of stresses will be established in the flange region. One of it is radial stress. It usually occurs radial outward from the side of the cup to the outer side of the blank material. Another is hoop stress. It can be generated in the flange. It is compressive in nature and perpendicular to the radial lines drawn to the side of the blank from the job axis. It is either parallel to the blank circumference or tangential to it. Inside of the blank material, these two stresses will be created by the use of punch force.

1. Introduction

Hydro-forming deep drawing process is one of the most important cold working processes. It is utilized to manufacture different shapes with uniform thickness of the deep drawn cups. In this process pressure of fluid is important parameter. The quality of cup without any failures such as fractures and wrinkles is only depends on this fluid pressure. In this process fluid pressure is applied on either sides of the blank of n-sided polygonal blank material. Before the deep drawing operation the hydraulic fluid is placed in between the punching Chamber and die cavity. These two connected by means of bypass Path in the die shown in the Figure (1) Some of the empty place is provided in between surface of die and blank holder. The gap is used to easy flow of fluid and moment of the blank. While deep drawing operation taking place punch moves towards die cavity, the hydraulic fluid pressure in the die cavity slowly increases to maximum value.

Then fluid pass through the bypass path and enter on the peripheral surface of the blank. On the either side of blank moving fluid film will be generated. When the pressurized fluid film moves either side of the blank surface shear force will be generated. The generated shear force is always proportional to the velocity gradient of the viscous fluid. It is also depends on the viscosity of the fluid. In this process subjected to equal shear forces by the effect of fluid pressure on both sides of the blank, the blank moves to the center of the gap. The blank is less than the height of the fluid film.

2. Literature Survey

Yossifon and Tirosh (1985) in this article, formulated and evaluated by considering classical plasticity theory, on the presumption of plane strain tensile collapse, rapture by tensile instability in hydro-forming deep drawing. Recommended for practical application are different opening areas with upper and lower limits explored by fluid pressures. Yossifon and Tirosh (1988) experiments were performed and a mathematical model for the deep drawing process for hydro-forming was created. They developed acceptable operating fluid pressure paths in their experimental work to predict the deep sections taken from the failures. Zhang and Danckert (1998) in this article, the basic principle of deep drawing phase hydro-forming is clarified. Principles and attributes are applied. In order to achieve better deep-drawn products with an exceptionally smooth surface finish and improved mechanical properties, theoretical and empirical simulations are described.

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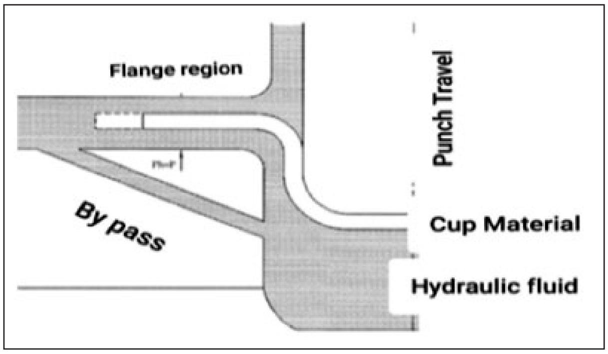

Fig. 1. Hydro-forming deep drawing process.

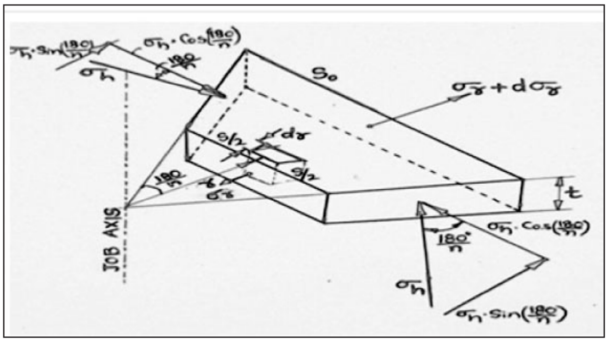

Fig. 2. Stresses developed in the flange area.

Thiruvarudehelvan and Lewis (1999) theoretical study of the hydro-forming deep drawing system with continuous hydraulic fluid pressure was introduced. Theoretical findings are contrasted with the experimental work in this paper and lower and upper limits for the pressure of the hydraulic fluid are also discussed. Thiruvarudehelvan and Travis (2003) the benefits and demerits of the hydro-forming deep drawing princess were explored by considering numerous hydraulic pressure variations. An expanded feature of manufacturing explored and addressed several methods. Lang et al. (2004) a new approach has been studied in the phase of hydraulic deep drawing. They exerted radially directed pressure force along rim of the blank material. They had performed an experimental study with aluminium material and the finite element analysis confirmed the results. Lang et al. (2005) in this paper extended the same work presented in the Lang et al. (2005) paper by considering numerical analysis. Studied the hydraulic deep drawing mechanism assisted by radial directed pressure and effect of process parameters. Abedrabbo et al. (2005) presented the wrinkling behavior of 6111-T4 aluminium alloy during sheet hydro-forming process. In this paper numerical, experimental analysis is done and finite element method is done for the validation of the experimental work. Udaykumar et al. (2017) studied the hoop stress behavior of magnesium alloys for cylindrical cup flange under hydro-forming deep drawing operation considering viscous phenomenon of hydraulic fluid.

3. Methodology

In this present paper mathematical modeling of hoop stress, the processing methodology considered to evaluate the hoop stress distribution and behavior of AA1100, galvanized commercial steel in hydro-forming deep drawing. Some of the notations used in this modeling are $\boldsymbol{\sigma_r}$ = Radial Stress, $\boldsymbol{\sigma_h}$ = Hoop Stress, $\boldsymbol{\sigma_y}$ = Yield stress of the blank material, S_0 = Side of the blank, S = Side of the flange, h = Height of the gap, t = Thickness of the blank, u = Fluid velocity, $\boldsymbol{\mu}$ = Viscosity of the fluid, $\boldsymbol{\tau}$ = Shear stress. On applying force equilibrium $1/n^{th}$ part of in the flange region, the different forces acting are shown in Figure.2.

On applying force equilibrium along radial direction, we get

$$\sigma_r.S.t-(\sigma_r.d\sigma_r).S_0.t-2.\sigma_h.\sin(\tfrac{180}{n}).\left(\frac{dr}{\cos(\frac{180}{n})}.t\right)+2.\tau.\tfrac{1}{2}.(S_0+S).dr=0$$

$$S_0=2.(r+dr).\tan(\frac{180}{n})$$

$$S=2.r.\tan(\frac{180}{n})$$

$$(\sigma_r+\sigma_h).dr+r.d\sigma_r=\frac{2.\tau}{t}.r.dr \qquad \text{............ (1)}$$

Where σ_r, σ_h are principal stresses acting on the elemental flange on applying Tresca's yield criteria.

$$\sigma_r-\sigma_h=\sigma_y$$

$$\sigma_r+\sigma_h=\sigma_y$$

(Where σ_h is compressive hoop stress)

$$\sigma_r=\sigma_y-\sigma_h \qquad \text{....................................(2)}$$

$$d\sigma_r=d\sigma_y-d\sigma_h \qquad \text{....................................(3)}$$

$$\sigma_y.dr+r.d\sigma_y-r.d\sigma_h=\frac{2.\tau}{t}.r.dr$$

$$r.d\sigma_h=\sigma_y.dr+r.d\sigma_y-\frac{2.\tau}{t}.r.dr$$

$$d\sigma_h=\sigma_y.\frac{dr}{r}+d\sigma_y-\frac{2.\tau}{t}.dr \qquad \text{...................(4)}$$

On integrating both sides of the above equation.

$$\int d\sigma_h = \sigma_y . \int \frac{dr}{r} + \int d\sigma_y - \frac{2.\tau}{t} . \int dr + C \qquad \text{...... (5)}$$

$$\sigma_h = \sigma_y . In[r] + \sigma_y - \frac{2.\tau}{t} . r + C \qquad \text{........... (6)}$$

Where C is constant of integration, it can be evaluated by applying boundary conditions at r=$R_{0,}\sigma_r = 0$ then $\sigma_h = \sigma_y$

$$C = \frac{2.\tau}{t} . R_0 - \sigma_y . In[R_0]$$

On substituting above C value in the equation Eq(6)

$$\sigma_h = \sigma_y . [1 - In\left[\frac{R_0}{r}\right] + \frac{2.\tau}{t} . (R_0 - r) \qquad \text{.................(7)}$$

In terms of n- sided polygonal side of blank an side of cup

$$\sigma_h = \sigma_y . [1 - In\left[\frac{S_0}{S}\right] + \frac{\tau}{t} . \left(\frac{S_0 - S}{\tan\left(\frac{180}{n}\right)} \right) \qquad \text{.............(8)}$$

The effect of viscosity in the hoop stresses of the blank material

$$(dy)_1 = (dy)_2 = dy = \frac{h - t}{2}$$

but $\tau_1 = \tau_2$

by following Newton's law of viscosity, the shear stress developed either sides of the blank are

$$\tau_1 = \mu . \left(\frac{du}{dy}\right)_1 = \tau$$

$$\tau_1 = \mu . \left(\frac{du}{dy}\right)_2 = \tau$$

Total shear stress developed in the blank will be

$$\tau_{Blank} = (\tau_1 + \tau_2) = 2.\tau = 2.\mu . \left(\frac{du}{dy}\right) = 2.\mu . \left(\frac{u - 0}{\frac{h - t}{2}} \right) = \frac{4.\mu.u}{h - t} \qquad \text{................(9)}$$

Now we can evaluate the hoop stresses in terms of viscosity

$$\sigma_h = \sigma_y . [1 - In\left[\frac{S_0}{S}\right] + \frac{2.\mu.u}{(h - t) . t} . \left(\frac{S_0 - S}{\tan\left(\frac{180}{n}\right)} \right) \qquad \text{....(10)}$$

3.1 Materials

3.1.1 AA1100

AA 1100 is one of the widest used alloys, with excellent forming properties in annealed states and may be suitable for bending, spinning, drawing, stamping, roll forming, and many other applications. Composition of AA1100 shown in below Table 1.

Table 1
Composition of AA1100.

AA 1100	Al	Si	Fe	Cu	Mn	Zn	Others Total
	99.00	0.95 Si+ Fe		0.05-0.20	0.05	0.10	0.15

Table 2
Composition of galvanized commercial steel.

Galvanized Commercial steel	Fe	C	Cr	Cu	Mn	Al
	98.7	0.016	0.03	0.07	1.06	0.09

Table 3
Material properties of AA1100, galvanize commercial steel.

Yield, Tensile Strength and Ductility Values for Al-1100 and Galvanized Commercial Steel			
Material	**Yield Strength**	**Tensile Strength**	**% Elong**
	MPa (ksi)	**MPa (ksi)**	
Aluminum Alloy 1100 Annealed (O Temper)	34 (5)	90 (13)	40
Galvanized Commercial Steel (hot-dipped)	330 (48)	59 ksi	28

Table 4
Variation of hoop stresses in flange of AA1100, galvanize commercial steel n-sided cup.

Side length of flange, S (mm)	Hoop Stress in AA1100 (MPa)	Hoop Stress in Galvanized Commercial Steel (MPa)
40	10.434	101.261
45	14.437	140.129
50	18.019	174.898
55	21.26	206.351
60	24.219	235.064
65	26.94	261.479
70	29.459	285.934
75	31.805	308.702
80	34	330

3.1.2 Galvanized commercial steel

Galvanized commercial steel is a form of steel made by dipping in molten zinc. Steel is covered with a zinc coating to protect steel from rusting. Steel starts to corrode when a scratch occurs on the surface. Composition of galvanized commercial steel shown in below Table.2

Mechanical Properties for hot-dipped galvanized commercial steel are Yield Strength: 48 ksi, tensile Strength: 59 ksi, Elongation: 28%, Hardness: 62 RB. Mechanical properties of AA1100, Galvanized Commercial Steel materials shown in the following Table.3

The hoop stress distribution in the flange of the blank material during hydro-forming deep drawing of n- Sided polygonal cup is given by Eq.(10)

$$\sigma_h = \sigma_y . [1 - \mathrm{In}\left[\frac{S_0}{S}\right] + \frac{2.\mu.u}{(h-t)\,.\,t}.\left(\frac{S_0 - S}{\tan\left(\frac{180}{n}\right)}\right)$$

4. Results and Discussions

By following process parameters and yield strength values of AA1100, Galvanized Commercial Steel metals are considered for the evaluation of the hoop stress in hydro-forming deep drawing of n-Sided polygonal cups. S_0=80mm, S=40mm to 80mm, μ=0.081 N - sec/m^2, u=10m/s, h=14mm, t=3mm, Material yield strength values: $(\sigma y)_{AA1100}$=34MPa, $(\sigma y)_{\text{Galvanize commercial steel}}$ = 330MPa .

With the variation of the flange sizes from S=40mm to S_0=80mm hoop stress developed in the n-sided polygonal flange can be estimated mathematically. Those values shown in the above Table 4.

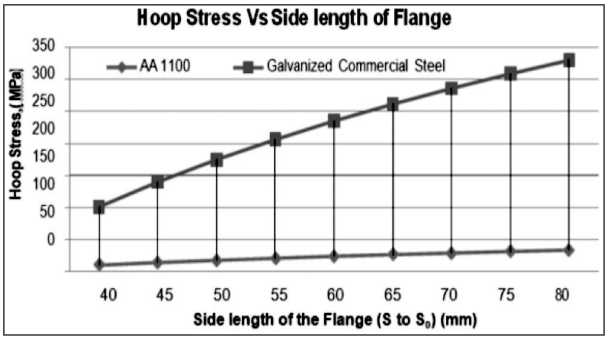

Fig. 3. Hoop stress variation in flange of n-sided cup.

From the results obtained, the hoop stresses of non ferrous and ferrous materials presented in Figure.3. It can be said that the hoop stress is increases in the flange with the increase of side length of the flange. Hoop stress value depends mostly on the yield stress of the material and other process parameters. If the yield stress value is more the hoop stress developed will be more. In the aluminium AA1100 blank material minimum hoop stress generated at the inner side of the flange is 10.434MPa at S=40mm, maximum hoop stress generated at the outer side of the flange is 34MPa at S_0=80. In the galvanized commercial steel blank material minimum hoop stress generated at the inner side of the flange is 101.261MPa at S=40mm, maximum hoop stress generated at the outer side of the flange is 330MPa at S_0=80. Among these two materials for any side length of flange in between S=40mm and S_0=80mm, the non ferrous aluminium AA1100 blank material is having lower hoop stress value; galvanized commercial steel is having greater hoop stress value. For the same side length of n-sided polygonal cups with increasing number of sides (varying volumes) i.e. n=3, 4, 5, experiences constant hoop stress values in the flange region.

5. Conclusions

1. Hoop stress is depends on the process parameters. It mostly depends on yield strength of material; if its yield strength value of blank material is more then hoop stress developed in the flange region will be more vice-versa.
2. The hoop stress developed in galvanized commercial steel is greater than AA1100. The hoop stress value at the inner side length S=40mm of the flange is 101.261MPa and at the outer side length S=80mm of the flange is 330MPa.
3. The hoop stress developed in AA1100 is smaller than galvanized commercial steel. The hoop stress value at the inner side length S=40mm of the flange is 10.434MPa and at the outer side length S=80mm of the flange is 34MPa.
4. As the side of the flange increases to blank size the hoop stress generated increases linearly in the flange up to maximum value of yield strength of material. For the same side length of the blank the maximum hoop stress developed in the AA1100 material is 34 MPa, maximum hoop stress developed in the galvanized commercial material is 330MPa.
5. The hoop stresses generated in the hydro-forming deep drawing process are responsible to increase the formability of the blank material in the flange region.

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References

Abedrabbo, N., Zampaloni, M. A., & Pourboghrat F. (2005). Wrinkling control in aluminum sheet hydroforming. *International Journal of mechanical Sciences*, 47(3), 333-358. https://doi.org/10.1016/j.ijmecsci.2005.02.003

Lang, L., Danckert, J., & Nielsen, K. B. (2004). Investigation into hydrodynamic deep drawing assisted by radial pressure: Part I. Experimental observations of the forming process of aluminum alloy. *Journal of Materials Processing Technology,* 148(1), 119-131. https://doi.org/10.1016/j.jmatprotec.2004.01.053

Lang, L., Danckert J., & Nielsen, K. B. (2005). Investigation into hydrodynamic deep drawing assisted by radial pressure: Part II. Numerical analysis of the drawing mechanism and the process parameters. *Journal of Materials Processing Technology,* 166(1), 150-161. https://doi.org/10.1016/j.jmatprotec.2004.08.015

Thiruvarudchelvan, S., & Travis, F. W. (2003). Hydraulic-pressure enhanced cup-drawing processes - An appraisal. *Journal of Materials Processing Technology*, 140(1-3), 70-75. https://doi.org/10.1016/S0924-0136(03)00726-X

Thiruvarudehelvan, S., & Lewis, W. (1999). A note on hydro-forming with constant fluid pressure. *Journal of Materials Processing Technology*, 88(1), 51-56. https://doi.org/10.1016/S0924-0136(98)00378-1

Uadaykumar, R., Ravinder Reddy, P., & Sitaramaraju, A. V. (2017). Determination of Hoop Stresses in Hydro-forming Process. *Materials today proceedings*, 4(8), 9133-9140. https://doi.org/10.1016/j.matpr.2017.07.269

Yossifon, S., & Tirosh, J. (1985). Rupture instability in hydro-forming deep-drawing process. *International Journal of Mechanical Sciences*, 27(9), 559-570. https://doi.org/10.1016/0020-7403(85)90072-4

Yossifon, S., & Tirosh, J. (1988). On the Permissible Fluid-Pressure Path in Hydro-forming Deep Drawing Processes-Analysis of Failures and Experiments. *Journal of Engineering for Industry,* 110 (2), 146-152. https://doi.org/10.1115/1.3187863

Zhang, S. H., & Danckert, J. (1998). Development of hydro-mechanical deep drawing. *Journal of Material Processing Technology,* 83(1-3), 14-25.https://doi.org/10.1016/S0924-0136(98)00039-9

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