PROCESS CAPABILITY (C_P AND C_{PK}) STUDY OF CNC MACHINES

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Abstract: *The accuracy of a machine tool is generally assessed in accordance with the relevant BIS/ISO standards, which serves mainly to provide information concerning the geometrical and other accuracies, as a rule with the machine in unloaded condition. Besides these tests, there are other factors important while assessing the quality and acceptance of the machine tool, based on of its capability in machining, which is considered effective and useful to the seller and buyer of the machine. BIS/ISO:230 standard define the machine tool acceptance based on its testing for geometrical errors and other performance parameters, whereas ISO:26303, ISO:22514 and VDI-DGQ:3441 standards provide a procedure for the acceptance of a machine tool based on of its capability in machining. The statistical verification process described in these standards is applicable to all machine tool categories, and it is necessary to follow typical test pieces and machining conditions, as per guidelines indicated in the VDI-DGQ standard.*

ISO recommends to calculate C_p/C_s using standard deviation by normal method, whereas VDI-DGQ recommends use of Range method, and trend to be corrected to estimate for C_p, if observed. ISO standard defines Cpk, based on upper and lower tolerance limits, and also method for surface finish like errors, including the permissible values. This document intends to provide consolidated information how the C_p/C_{pk} is defined in ISO and also the method of calculation by an example. There is lot confusion in the minds of industry, to calculate the C_p/C_{pk} and the recommended permissible values for the machine performance. The purpose of this document is to make the industry people (Manufacturer's and users) to understand the calculation of C_p, well talked by the people.

The purpose of this study is to bring out the differences defined between these various standards, explain the method of estimating the process capability index with an example, and also provide source for the recommended permissible values as per relevant standards, to help industry partners in understanding and the calculation of the process capability index.

Key words: *C_p, C_{pk}, Testing, Process Capability, Tolerance*

1. INTRODUCTION

The quality of the component machined is the decisive for the operational accuracy of a machine, determined from the dimensional variation of the test pieces under specific machining conditions using statistical methods.

The dimensional variations arise while machining of components gives a direct indication of operational scatter or manufacturing accuracy. The statistical methods establish whether the operational scatter of the machined component dimension lies with in the manufacturing tolerance of the component. The capability of the process/

machine indicates the suitability of process/ machine to achieve the particular tolerance limit from the defined process for a particular job.

BIS/ISO:230 standards define the machine tool acceptance based on its testing for geometrical errors and other performance parameters, whereas ISO:26303, ISO:22514 and VDI-DGQ:3441 standards provide a procedure for the acceptance of a machine tool based on of its capability in machining. ISO:22514 standard define the procedure by estimating the Process Capability index (C_p) and ISO:26303 standard define the procedure by estimating the Short-term Capability index (C_s), considering the standard deviation (of the measured dimensional deviations on a specified workpiece. The Capability index C_p and C_s defined in these ISO standards represent the same and the calculation is same for the estimation of these indexes. The same is also referred as Machine capability index (C_m) by few users. The statistical verification process described in these standards is applicable to all machine tool categories.

Whereas, the VDI-DGQ:3441 standard define the procedure for by measurement of Operational accuracy/scatter ($6S_R$) on a specified work piece, and recommends the limiting values for the Operational accuracy/scatter ($6S_R$as a percentage distribution of work piece tolerance. This standard does not indicate anything about Capability index C_p.

As ISO standards do not provide details about the workpiece and machining details, it is suggested to follow typical test pieces and machining conditions as per guidelines indicated in VDI-DGQ standards to carry out the process capability study. The method of calculation is same for turning, milling, grinding, except he workpiece/machining details.

2. PROCESS CAPABILITY STUDY (REF: ISO STANDARD)

For the estimation of the process capability study, ISO standard recommends to use 100 components, and in some machines, which have very slow cycle times, it will be necessary to proceed with available data, and ISO 22514 standard recommends the minimum number as 30. Ensure an uninterrupted machining takes place, under normal operating conditions. This will include any warm-up time for the machine necessary to bring it up to its usual operating condition. If the machine is stopped during the study for whatever reason, either re-run the study again or analyse the data collected. Under no circumstance should less than 30 results be used. The Process Capability index (C_p) is the most widely used Process Capability index in the manufacturing industry, and the following steps describe the calculation of Process Capability index with reference to ISO:26303.

The Process Capability is indicated as the ratio of Tolerance (T) to dimensional spread of 6 (Six times standard deviation), and signifies that 99.76% of the machine caused dimensional variations are equal to Tolerance specification.

The C_p index is the most widely used Process Capability index, calculated only when the mean is centered between the specification limits (U and L) and the C_{pk} is the minimum Critical Process Capability Index, calculated when mean is not centered between specification limits (U and L).

The following symbols are used for calculation of process capability.

C_p = Process Capability Index

C_{pk}= Critical Process Capability index, which is minimum of C_{pkL} and C_{pkU}

C_{pkL}=Critical Process Capability index corresponding to Lower Tolerance limit

C_{pkU}=Critical Process Capability index corresponding to Upper Tolerance limit

U = Upper Specification Limit or Upper tolerance limit

L = Lower Specification Limitor Lower tolerance limit

T = Total Tolerance (U-L)

n = Number of components in one sub-group

m = Number of sub-groups

$\bar{x}_m$ = Average mean of sub-group

$= \sum x_i/n = (x_1+x_2+x_3+x_4+x_5)/n_1$

$\bar{\bar{x}}$ = Total average of m samples ($\bar{\bar{x}}=\sum\bar{x}/m$),

Total mean Value of all components

s = Standard deviation by normal method

$= \sqrt{\sum(\mathrm{xi}-\bar{\mathrm{x}})2/(\mathrm{n}-1)}$

$\bar{s}$= Average of Standard deviation of m samples

$\bar{\sigma}$ =Estimated standard deviation = $\bar{s}$/m

3. CALCULATION OF PROCESS CAPABILITY INDEX AS PER ISO: 26303

Test piece dimensions, cutting tool and machining conditions are selected as per the relevant VDI-DGQ standard for various machining processes (Turning, Milling, Drilling and Grinding). A batch of 50 test pieces/components are machined at the recommended machining conditions, and the components are numbered in serial order (1 to 50). The total components are divided into 10 sample sub-group sets (m=10) of each sub-group with 5 components (n=5), and the measured dimensions are entered serially in the table as indicated below.

n=1-5 in sample set m_1,

n=6-10 in sample set m_2,

n=7-15 in sample set m_3,

n=16-20 in sample set m_4,

n= 21-25 in sample set m_5,

n= 26-30 in sample set m_6,

n= 31-35 in sample set m_7,

n= 36-40 in sample set m_8,

n= 41-45 in sample set m_9,

n= 46-50 in sample set m_{10}

Next The average mean ($\bar{x}$) of each sub-group, Standard Deviation (s), and Trend per piece (Tp) is calculated and entered in the following table.

The increase in average mean ($\bar{x}$) from m_1 to m_{10} indicates the presence of Trend and the Trend is need to be further corrected to estimate the process capability.

Correction for Trend

Trend corrected values for all components is calculated as below, and entered in the table below:

Total Trend of sub-groups (T) = $(\bar{x}m10\text{-}\bar{x}m1) = (16.4\text{-}1.6) = 14.8$

Trend per piece (T_p) = $(\bar{x}m10\text{-}\bar{x}m1)/49 = (16.4\text{-}1.6)/49 = 0.30$

Trend correction is calculated using the formula: $x_{iT} = x_i\text{-}(i\text{-}1) *T$

Sub-group number (m_1 to m_{10})	1	2	3	4	5	6	7	8	9	10
Test piece number in sub-group (n_1 to n_5)	X_j (µm)									
1	0	2	4	5	6	7	9	11	10	14
2	1	2	3	6	5	6	8	10	11	13
3	1	3	5	9	6	10	12	13	13	16
4	2	2	6	6	7	9	11	14	15	17
5	4	6	8	10	11	13	16	19	18	22
Average mean ($\bar{x}$), $\bar{x}$m1 to $\bar{x}$m10	1.6	3	5.2	7.2	7	9	11.2	13.4	13.4	16.4
Standard Deviation (s)	1.52	1.73	1.92	2.17	2.35	2.74	3.11	3.51	3.21	3.51
Average Standard Deviation($\bar{s}$)	2.58									
Trend per piece (T_p) =$(\bar{x}_{m10}\text{-}\bar{x}_{m1})/49$	0.30									

Where, x_{iT}=i[th] Trend corrected measurement, x_i= i[th] measurement (not trend corrected)

AFTER TREND CORRECTION (Trend per piece=0.30)										
Sub-group number (m1 to m10)	1	2	3	4	5	6	7	8	9	10
Test piece number in one sub-group (n1 to n5)	X_j (µm)									
1	0.00	0.50	1.00	0.50	0.00	-0.50	0.00	0.50	-2.00	0.50
2	0.70	0.20	-0.30	1.20	-1.30	-1.80	-1.30	-0.80	-1.30	-0.80
3	0.40	0.90	1.40	3.90	-0.60	1.90	2.40	1.90	0.40	1.90
4	1.10	-0.40	2.10	0.60	0.10	0.60	1.10	2.60	2.10	2.60
5	2.80	3.30	3.80	4.30	3.80	4.30	5.80	7.30	4.80	7.30
Average mean ($\bar{x}$)	1.00	0.90	1.60	2.10	0.40	0.90	1.60	2.30	0.80	2.30
Standard Deviation (s)	1.08	1.42	1.51	1.85	1.98	2.34	2.72	3.09	2.74	3.09

The following values are estimated for the trend corrected errors (Calculations as described below:

1. Total Average ($\bar{\bar{x}}$)	1.39
2. x_{max}	7.30
3. x_{min}	-2.00
4. Range (R)=(X_{max}-X_{min})	9.30
5. x_{max}- $\bar{\bar{x}}$	5.91
6. $\bar{\bar{x}}$ - x_{min}	3.39
7. Range value-Percentage of Tolerance (R_s)=(R/T) *100	31%
8. Average Standard Deviation ($\bar{s}$)	2.18
9. Estimated Std. Deviation ($\bar{\sigma}$) = ($\bar{s}$)/0.94	2.32
10. 6σ	13.93
11. 3σ	6.96
12. USL- $\bar{\bar{x}}$	13.61
13. $\bar{\bar{x}}$ - LSL	16.39
14. Process Capability Index (C_p)=T/6σ	2.15
15. Critical Process Capability Index (Cp_{kU})= (USL- $\bar{\bar{x}}$)/3σ	1.95
16. Critical Process Capability Index (Cp_{kL})= ($\bar{\bar{x}}$-LSL)/3σ	2.35
17. Critical Range Value-Percentage of Upper Tolerance (R_{sku}) = (x_{max}- $\bar{x}$)/(U-$\bar{x}$). Roughness value of 10µm	69%

3.1 FOR BIDIRECTIONAL SPECIFICATION OF TOLERANCE

Assume the required bi-directional specification of tolerance as 30µm (U=+15 µm, L=-15 µm)

a) <u>Process capability</u> (C_p) is calculated by considering mean values of Upper tolerance limit(U) and Lower tolerance limit(L)

$C_p = T/6\bar{\sigma} = (U-L)/6\bar{\sigma} = (15+15)/13.93 = 2.15$

b) <u>Critical Process capability</u> (C_{pk}) is calculated as the minimum value of Critical Process Capability Index (C_{pkU} and (C_{pkL}), considering the Upper (U) and Lower (L) tolerance limits.

C_{pk}= min (C_{pkU} and C_{pkL}), where, $C_{pkU} = (U - \bar{\bar{x}})/3\bar{\sigma}$ and $C_{pkL} = (\bar{\bar{x}} - L)/3\bar{\sigma}$

$C_{pkU} = (U - \bar{\bar{x}})/3\bar{\sigma} = (15-1.39)/(3x2.32) = 1.95$

$Cp_{kL} = (\bar{\bar{x}}-L)/3\bar{\sigma} = (1.39+15)/(3x2.32) = 2.35$

As the process average ($\bar{\bar{x}}$) is not at the center of the tolerance, the process capability index is defined as C_{pk}=1.95, which is minimum of C_{pkU} and C_{pkL}.

3.2 For Unidirectional Specification Of Tolerance

Assume the required unidirectional specification of tolerance as 0-30µm (U=+30 µm, L=0.0 µm).

<u>Process capability</u> (C_p) is calculated by considering the Upper (U) or Lower (L) tolerance limits. The calculation is similar to the procedure applied in the case of two-sided tolerance limits.

C_{pk}= min (C_{pkU} and C_{pkL}), where, $C_{pkU} = (U - \bar{\bar{x}})/3\bar{\sigma}$ and $C_{pkL} = (\bar{\bar{x}} - L)/3\bar{\sigma}$, as the case may be.

$C_{pkU} = (U - \bar{\bar{x}})/3\bar{\sigma} = (30-1.39)/(3x2.32) = 28.61/6.96 = 4.11$

$Cp_{kL} = (\bar{\bar{x}}-L)/3\bar{\sigma} = (1.39+0)/(3x2.32) = 1.39/6.96 = 0.20$

As the process average ($\bar{\bar{x}}$) is not at the center of the tolerance, the process capability index is defined as C_{pk}=0.20, which is minimum of C_{pkU} and C_{pkL}.

3.3 For Roughness & Form Tolerance

Roughness and form tolerances (cylindricity, circularity, taper etc.) are defined only with Upper limit of tolerance, and standard specify to estimate the critical range value (R_{skU}). The critical range value (R_{skU}) is defined as maximum percentage of Tolerance(T).

Assuming, Surface roughness as 10µm, R_{skU} is calculated as below:

$R_{skU} = (x_{max}- \bar{\bar{x}})/(U-\bar{\bar{x}})$ *100 = ((5.91/(10-1.39)) *100 = 69% of T

3.4 RECOMMENDED PERMISSIBLE VALUES AS PER ISO 26303

Note: The manufacturer/supplier and the user shall negotiate which of the below two characteristic values is used for acceptance, for the defined workpiece and machining conditions.

Process/Feature	C_p	C_{pk}	Minimum of R_s and R_{sk}
Normal processes or features (bi-directional tolerance)	≥1.67	≥1.67	—
In-process measurement control	—	—	≤100% of T
Roughness values (there is only an upper limit)	—	—	≤80% of T
One-sided limited tolerance	—	≥1.67	≤60% of T
Other special processes or features (e.g. meas. control)	≥1.67	≥1.67	≤60% of T

4. ESTIMATION OF OPERATIONAL SCATTER AS PER VDI/DGQ:3441: (for the same data as above)

A batch of 50 test pieces/components are machined at the recommended machining conditions, the components are numbered in serial order (1 to 50). The total components are divided into 10 sample sub-group sets (m=10) of each sub-group with 5 components (n=5), and the measured dimensions are entered serially in the table as indicated below.

n=1-5 in sample set m_1, n=6-10 in sample set m_2, n=7-15 in sample set m_3,

n=16-20 in sample set m_4, n= 21-25 in sample set m_5, n= 26-30 in sample set m_6,

n= 31-35 in sample set m_7, n= 36-40 in sample set m_8, n= 41-45 in sample set m_9,

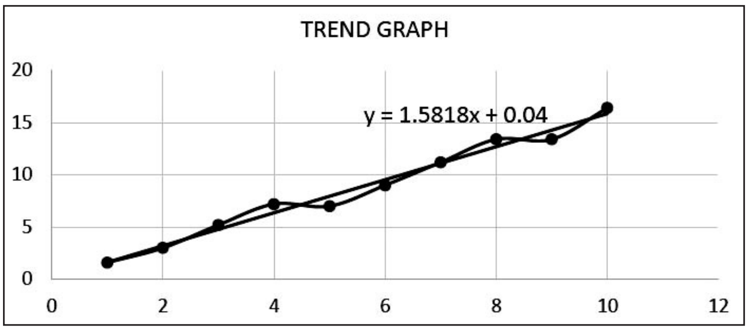
n= 46-50 in sample set m_{10}

Next The average mean ($\bar{x}$) of each sub-group, Standard Deviation (s), and Trend per piece (Tp) is calculated and entered in the following table. The Mean ($\bar{x}_m$), mean range ($\bar{R}$), Standard deviation (S_R) and Operational scatter (A_S) is calculated and entered in the following table of calculations.

Sample set number (m)	1	2	3	4	5	6	7	8	9	10
Test piece number in one sample set (n)	X_j (µm)									
1	1	2	4	5	6	7	9	11	10	14
2	0	2	3	6	5	6	8	10	11	13
3	1	3	5	9	6	10	12	13	13	16
4	2	2	6	6	7	9	11	14	15	17
5	4	6	8	10	11	13	16	19	18	22
$\sum x_i$	8	15	26	36	35	45	56	67	67	82
Mean, $\bar{x}_m = \sum x_i/n$	1.6	3	5.2	7.2	7	9	11.2	13.4	13.4	16.4
Range, $R = X_{i\,Max} - X_{i\,Min,}$ (µm)	4	4	5	5	6	7	8	9	8	9
Average Range ($\bar{R}$)	6.5									
Operation Scatter ($6S_R$)	16.77									

The increase in average mean ($\bar{x}$) from m_1 to m_{10} indicates the trend and the trend is corrected to estimate the process capability.

<u>Correction for Trend</u>: Trend graph is plotted for mean of the sample set ($\bar{x}_m$)against sample set (m), and the trend equation is arrived from the trend graph.From the trend equation, the total trend (T) is calculated by extending the sample set by ONE set (i.e. for 11 sample sets), then trend per component (T_p) is calculated as T/(n-1).



n1	2	3	4	5	6	7	8
dn	1.128	1.693	2.059	2.326	2.534	2.704	2.847

AFTER TREND CORRECTION (Trend per piece=0.30)										
Sub-group number (m1 to m10)	1	2	3	4	5	6	7	8	9	10
Test piece number in one sub-group (n1 to n5)	X_j (µm)									
1	0.00	0.40	0.80	0.20	-0.40	-1.00	-0.60	-0.20	-2.80	-0.40
2	0.68	0.08	-0.52	0.88	-1.72	-2.32	-1.92	-1.52	-2.12	-1.72
3	0.36	0.76	1.16	3.56	-1.04	1.36	1.76	1.16	-0.44	0.96
4	1.04	-0.56	1.84	0.24	-0.36	0.04	0.44	1.84	1.24	1.64
5	2.72	3.12	3.52	3.92	3.32	3.72	5.12	6.52	3.92	6.32
Average mean ($\bar{x}$)	0.96	0.76	1.36	1.76	-0.04	0.36	0.96	1.56	-0.04	1.36
Range (R)=(Xmax-Xmin)	2.72	3.68	4.04	3.72	5.04	6.04	7.04	8.04	6.72	8.04

Total Mean ($\bar{\bar{x}}$)	0.90
X_{max}	6.52
X_{min}	-2.80
Average Range ($\bar{R}$)	5.51
Statistical Constant d_n for 5 samples	2.326
Std. Deviation (S_R) = ($\bar{R}$)/2.326	2.37
Operation Scatter ($6S_R$)	14.21
$3S_R$	7.10
U - $\bar{\bar{x}}$	14.10
$\bar{\bar{x}}$ - L	15.90
Process Capability Index (C_p)=T/$6S_R$	2.11

The sample calculation of trend per component is calculated as below:

The average mean ($\bar{x}$) corresponding to 11th sub-group is calculated using the slope formula from the above graph, $\bar{x}$ = 1.5818x + 0.04

Hence, $\bar{x}$ at 1st sub-group is $\bar{x}$ = (1.518*1+0.04) = 1.62

$\bar{x}$ at 11th sub-group is $\bar{x}$ = (1.518*11+0.04) = 17.44

Total Trend of sub-groups (T) = (17.44-1.62) =15.8

T_p = Trend per component = T/(n-1) = (17.44/49) = 0.32

	With Trend	With Trend corrected
Operational scatter ($6S_R$)	60% of tolerance (T) i.e. C_P= (1/0.60) =1.67*	80% of tolerance (T) i.e. C_P= (1/0.80) =1.25*
Trend	20% of tolerance (T)	
Measurement	100% of tolerance (T)	20% of tolerance (T)
*by defining Cp as (Tolerance/6SR)		

Trend correction is calculated using the formula:

$x_{iT} = x_i-(i-1) *T$

Where, x_{iT}=ith Trend corrected measurement

x_i= ith measurement (not trend corrected)

With this relationship, every measured value X_i (X_1 to X_{50}) is to be corrected for the trend portion.

$X_{icorr} = X_i - X_{iT}$, and the measured values Xi_{corr} corrected for the trend portion, are entered in the table below in the machining sequence under.

R= Sub-group Range, R = $X_{i\,Max}$-$X_{i\,Min}$,

$\bar{R}$ =Average range of sub-group = $\sum R_j/m$

S_R=Standard deviation by range method =/d_n,

where, d_n is statistical constant depending on the number of pieces based on sub-group size.

The following values are estimated for the trend corrected errors:

The percentage distribution of workpiece tolerance, corresponding to 30µm tolerance:

with trend = $(6S_R/T)$ *100 = (16.77/30) *100 = 56%
with trend corrected= $(6S_R/T)$ *100 = (14.21/30) *100 = 47.3%

The Process Capability index (C_p) is calculated in the similar lines as defined in ISO standard, using Standard deviation by Range method for the comparison purpose, is presented below:

Critical Process Capability Index (C_{pkU}) = $(U-\bar{\bar{x}})/3S_R$ = 1.98

Critical Process Capability Index (C_{pkL}) = $(\bar{\bar{x}}-L)/3S_R$ = 2.24

As the process average ($\bar{\bar{x}}$) is not at the center of the tolerance, the process capability index is defined as C_{pk}=1.98, which is minimum of C_{pkU} and C_{pkL}.

4.1 RECOMMENDED PERMISSIBLE VALUES AS PER VDI/DGQ: 3441

Note: The following percentage distribution of workpiece tolerance is recommended by VDI/DGQ. The recommended permissible values are to be agreed between manufacturer/supplier for the acceptance of machine or the process, for the defined workpiece, machined under defined machining conditions.

REFERENCE STANDARDS

1. ISO:26303-2012: Short term capability of machining Process on metal cutting machine tools
2. ISO 22514-4 2016: Process Capability estimates and Performance measures
3. VDI/DGQ 3441: Statistical Testing of Operational and Positional Accuracy of Machine tools
4. VDI/DGQ 3442: Statistical Testing of the Operational Accuracy of Lathes
5. VDI/DGQ 3443: Statistical Testing of the Operational Accuracy of Milling Machines
6. VDI/DGQ 3444: Statistical Testing of the Operational Accuracy of Drilling Machines
7. VDI/DGQ 3445: Statistical Testing of the Operational Accuracy of Grinding Machines
8. Sheet-1: Fundamentals
9. Sheet-2: Plain Grinding Machines
10. Sheet-3: Centreless Grinding Machines
11. Sheet-4: Internal Grinding Machines
12. Sheet-5: Surface Grinding Machines ◘

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He is a graduate in Mechanical Engineering and Post-graduate in Design and Production of Machine Tools from Regional Engineering College (NIT), Warangal. He has more than 40 years of experience in Machine Tool Testing, Assembly, Design and Manufacturing Engineering.

He Specialized in: Testing of Machine Tools for their performance as per National and International standards, Improving Performance and Productivity of Machine Tools, Testing, Trouble shooting and Problem solving at site to achieve required machine accuracy and productivity, Consultancy on Specific aspects of Testing of machine Tools, Preparation of Test Protocols with acceptance criteria for Performance Evaluation of CNC machines, where the relevant Standards are not available.

He represents IMTMA as member of BIS Sectional Committees PGD-35 (Machine Tools, Machine Tool Elements and Holding Devices) and MED-40 (Manufacturing Machinery and their safety).