MATHEMATICAL MODELING OF SPINDLE DEFLECTION FOR MULTI SPINDLE AUTOMAT AND VALIDATION USING ANSYS SOFTWARE

[1]GS Pramod Krishna and [2]Tom Saju

[1]Central Manufacturing Technology Institute, Bengaluru
[2]Karunya Institute of Technology and Science, Coimbatore, Tamil Nadu
Email: [1]Pramod.cmti@gov.in

Abstract: *Motorized spindles are one of the main sub-systems in a high-speed machine tool. In this paper, work is carried out to estimate the effect of machining operations on spindle deformation of a multi spindle automat. Spindle is supported on four bearings and analytical deflection calculation has been carried out considering the spindle as a continuous beam having 3 supports. Cutting forces acting on the spindle is calculated mathematically. Spindle deflection due to the cutting force has been calculated and validated using ANSYS. From the results, it's clear that mathematical modeling deflection value is almost similar to ANSYS results. From the optimization of bearing span, results prove that change in bearing span doesn't have much effect on spindle deflection.*

Keywords: *Spindle Deformation, Finite element analysis, Solidworks, Ansys*

INTRODUCTION

The spindle system is one of the most important systems in a machine tool affecting its performance. So any improvement in the dynamic characteristics of the spindle can improve the working of the spindle. In this paper, we mainly discuss with the spindle deflection during turning operation for a multi spindle automat and FEA analysis of spindle.

W R Wang and C N Chang have carried out dynamic analysis and design of spindle–bearing system in the year 1994. Bearing preload, mass inserts on spindle and damping has been used as the parameters for his study. In this paper, he proved that by adjusting the fore mentioned parameters improvements can be made in computational experimental results. [1] Jaijun Feng, Chengche Li and Zhi Wu in their paper on the analysis of static and dynamic characteristics of spindle system discusses with the design of a three-dimensional finite element modal and static and dynamic analysis of spindle bearing system. In his paper critical speed of the system has been calculated and displacement frequency curve is plotted. [2] Yuzhong Cao and Yusuf Altintas in their paper titled general method for modeling of Spindle – Bearing system explains a method for spindle assembly consisting of the spindle shaft, angular contact ball bearings, and housings. This paper provides details of the mathematical modeling supported by experimental results and natural frequency, frequency response function of a spindle. [3] Asim Kutlu in his thesis on design and development of lathe spindle has provided a finely detailed design of spindle following the verification and optimization of the preliminary design. [5] E Abele, Y Atlantis and C Brecher in the paper gives detailed information about the main components of spindle units. Heat sources of spindle, heat transfer type, bearing types, overview and selection, lubrication and cooling mechanism are some of the topics discussed in this paper. [6] Ayush Anand and Himel Roy in the year 2018 published a paper titled Static and dynamic analysis of Lathe spindle using ANSYS, in which they explain about the dynamic analysis to optimize static modal and modal shapes. Transient analysis is performed in 6000 rpm and deformation is calculated. [7] Mathieu Ritou in his paper titled Influence of spindle condition on the dynamic behavior indentifies the major failure modes and their effects on FRF. Faulty bearings modify mode amplitudes and degraded HSK modifies the frequency of bending modes. [8] Siddesh K.B used a finite element modal to study the dynamic characteristics of the spindle and explains how it affects the cutting ability and productivity of the machine and provides a conclusion that machine performance can be improved by increasing the dynamic stiffness of the spindle. [9]

Mathematical modeling and comparing the results with finite element analysis help us to validate the deflection study for multi spindle automat. Spindle parameters used for spindle calculation include stiffness values for bearings, turning force calculation, bearing selection, and bearing span. In my work four bearing is selected as the support for the spindle and stiffness values of the bearings has been taken from SKF catalogs. After the calculation of the spindle deflection and validated using ANSYS, bearing span is varied and deflection of the spindle has been calculated.

MATHEMATICAL MODELING FOR SPINDLE DEFLECTION

Machining force calculation for turning operation is carried out and cutting force is calculated analytically. The effect of turning cutting force on spindle deformation is mathematically modeled and validated through finite element analysis in ANSYS.

Analytical calculation and preliminary design:

Cutting force, power calculation and preliminary design of the spindle system is discussed in the following section.

a) Machining force calculation:

During the machining operation, spindle experiences deformation due to cutting forces. Cutting force during turning operation that causes spindle deformation is calculated analytically. Here in this study GC4335 turning insert type is considered. GC4335 is of CVD coated carbide and insert specifications have been taken from Sandvik catalog. Workpiece material is ball bearing steel type with alloying element less than 5 percentage and hardness of 210HB. Recommended cutting conditions are given in table 1.

Table 1: Recommended Cutting Condition

Parameters	Units	Value
The Depth of cut, a	mm	1-3
Cutting feed, f	mm/rev	0.2-0.4
Specific cutting force, Kc	N/mm^2	1800
Cutting speed, V_c	m/min	100

Based on Sandvik cutting condition for the insert with 93^0 approach angle, 0.8 nose radius and 7^0 rake angle, calculation factor for axial and radial force are given as 0.55 and 0.3 respectively. Calculation factor values are taken from the HMT production catalogue and this calculation factor is used to find the radial and axial cutting forces.

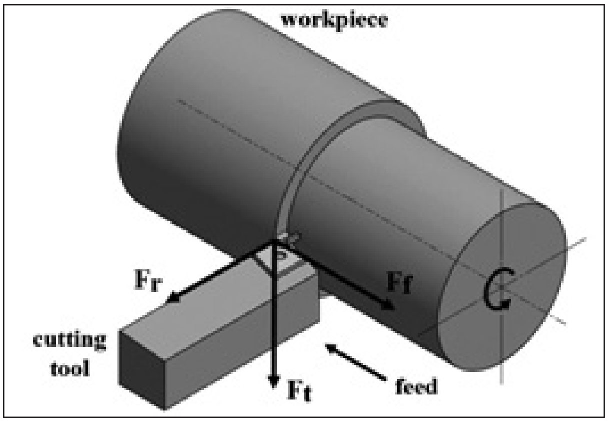

Fig 1. Cutting Forces During Turning Operation

Net power consumption:

$$P\ (\text{in kW}) = \frac{Vc\left(\frac{m}{min}\right)x\ a(mm)x\ f\left(\frac{mm}{rev}\right)xKc\ (N)}{60000}$$
$$= 1.5\ kW$$

Tangential cutting force:

$$Ft\ (\text{in N}) = \frac{P(W)}{Vc\ (\frac{m}{sec})}$$
$$= 900\ N$$

Radial cutting force, Fr = (0.3 x Ft)
= 270 N

Axial cutting force, Fa = (0.55 x Ft)
= 495 N

Resultant load acting on the work-piece:

$$Fc\ (\text{in N}) = \sqrt{Fr^2 + Ft^2}$$
$$= 940\ N$$

a) Preliminary design of spindle.

Design of spindle for multi spindle automat including the assembly of collet, drawbar, work piece, bearings, and rotor is modelled in SOLIDWORKS. The first set of bearing attached to the spindle is angular contact ball bearing in a tandem arrangement. The second bearing selected is also the same angular contact ball bearing. The angular contact ball bearing is selected as they can withstand both radial and axial load and provide high stiffness to the spindle. The cylindrical roller bearing is chosen as the third bearing due to its high radial stiffness.

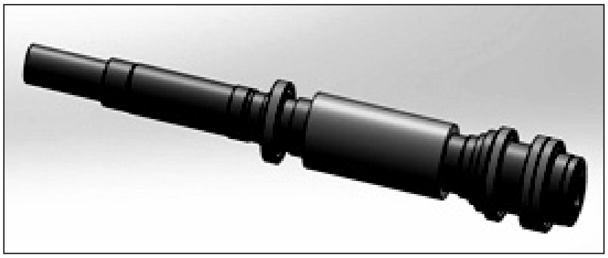

Fig 2. SOLIDWORKS Drawing of the Spindle with Bearings

The material of the spindle is structural steel and properties are in correlation with structural steel. Bearing specifications selected for modelling are given in table 2.

Table 2: Bearing Type and Dimensions

Bearing type	Dimensions
Angular contact ball bearing	71918 ACD, 90 X 125 X 18
Cylindrical roller bearing	NN 1014, 70 X 110 X 20

B) Spindle deflection calculation

Analytical calculation for spindle deflection is carried out. Figure 3 shows the Auto cad drawing of a spindle for multi spindle automat.

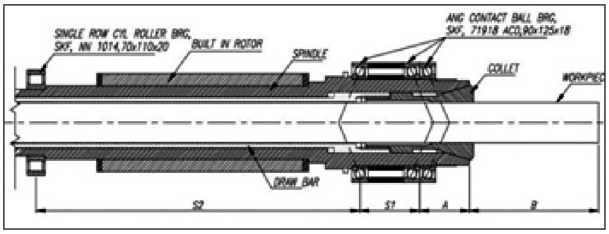

Fig 3. AUTOCAD Drawing of the Spindle

Here the spindle is considered as a continuous beam. Load carrying capacity of a continuous beam is more and mid span deflection will be less compared to other beams. In the present design, there are four supports and mathematical modeling is carried out considering the spindle as a continuous beam. First set of bearings is taken as a single support and hence spindle is considered as a continuous beam with three supports. Analytical calculation will be difficult as it involves inverse matrix calculations and tedious procedures.

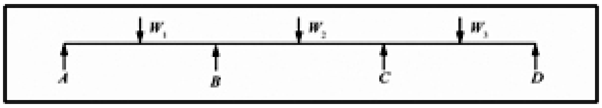

Fig 4. Continuous Beam

Here mathematical modeling for the spindle deflection calculation is carried out based on superposition principle. Superposition principle considers the spindle deflection calculation as two different cases. The first consideration is such that elastic beam- rigid support case and second consideration is like rigid beam – elastic support case. Total deflection is given by adding the two separate deflections for both the cases.

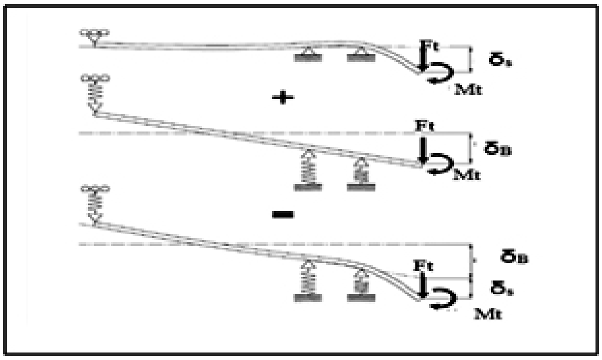

Fig 5. Superposition Principle

Total deflection of the spindle, $\delta_{SPINDLE} = \delta_S + \delta_B$

Recommended spindle conditions are given in table 3. All the cutting conditions for the spindle are selected based on operations carried out.

Table 3: Recommended Spindle Condition

Parameters	**Units**	**Values**
Overhung distance, a	mm	60
b	mm	140
Span distance, S1	mm	67
Span distance, S2	mm	360
The load applied, Ft	N	939.6275
Moment, Mt	Nmm	131547.85
Stiffness of the bearing, Ka	N/mm	790000
Stiffness of the bearing, Kb	N/mm	790000
Stiffness of the bearing, Kc	N/mm	850000
Inner dia, d	mm	55
Outer dia, D	mm	75

Note that the cutting force Ft is generated on the workpiece and far away from spindle but in this case, it is moved on to spindle tip together with a coupled moment Mt.

Mt = b x Ft
= 131548 N/mm

S_1 and S_2 are the bearing spans of the spindle and A is the overhung distance of the spindle. Constraints are applied to bearing span considering the rotor dimensions, such that:

Max possible span:
S1 ≤ 67 mm
S2 ≤ 385 mm

Min possible span
S1 ≥ 42 mm
S2 ≥ 360 mm

An optimization analysis will be done for the spindle to find the best possible bearing span and lowest spindle deflection.

Assumptions made for the superposition principle are:

- -Only cutting and reaction forces are experienced by the spindle.
- -Cutting forces are acting only in tangential - longitudinal direction.
- -Parts that are having an interference fit with the shaft are considered perfectly bonded and a part of the spindle.
- -Each bearing pairs are considered to be the support and 1st set of the bearing is considered as single support.

Condition 1: Elastic beam rigid support case:

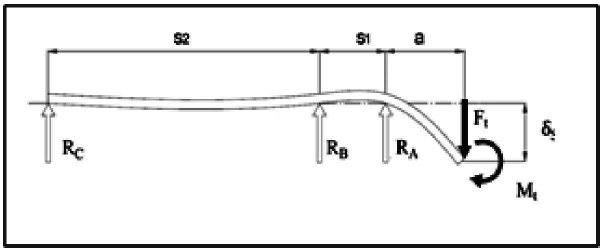

Fig 6. Elastic Beam – Rigid Support Case

$$R_A + R_B + R_C = F_t \quad \text{...........(1)}$$

$$R_B S_1 + R_C(S_1+S_2) + F_t a + M_t = 0 \quad \text{...........(2)}$$

Spindle system is shown in figure no 5. Here three unknown forces need to be solved and for that, deflection equations and boundary equations are being used. From equation (1) and (2) reaction forces R_B and R_C:

$$R_A = \frac{Ft(S1+S2+a) - R_A(S_1+S_2) + Mt}{S_2} \quad \text{..........(3)}$$

$$R_B = -\frac{Ft(S1+a) - R_A S_1 + Mt}{S_2} \quad \text{.......... (4)}$$

The loading function of the spindle is written using Macaluay and singularity function. The deflection function of the spindle can be obtained by integrating the loading function four times. The loading function of the shaft can be expressed as:

$$W_s = Ft(x-0)^{-1} + Mt(x-0)^{-2} - R_A(x-a)^{-1} - R_B(x-(S_1+a)^{-1} \quad (5)$$

Integrating the loading function will give the shear function:

$$V_s = -Ft(x-0)^0 - Mt(x-0)^{-1} + R_A(x-0)^0 + R_B(x-(S_1+a))^0 \quad (6)$$

Integrating again will give moment function of the shaft:

$$M_s = -Ft(x-0)^1 - Mt(x-0)^0 + R_A(x-a)^1 + R_B(x-(S_1+a)^1 \quad (7)$$

$$M_s = -Ftx - Mt + R_A(x-a)^1 + R_B(x-(S_1+a))^1 \quad (8)$$

Now, using moment and deflection equation:

$$EI\frac{d^2V_s}{dx^2} = M_S(x) \quad (9)$$

Substituting the moment function in equation no (9) and integrating it will give the slope function:

$$EI\frac{dV_s}{dx} = -\frac{Ftx^2}{2} - Mt + \frac{R_A}{2}(x-a)^2 + \frac{R_B}{2}(x-(S_1+a))^2 + C1 \quad \text{...(10)}$$

Integrating the above equation:

$$EIV_S = -\frac{Ftx^3}{6} - \frac{Mtx^2}{2} + \frac{R_A}{6}(x-a)^3 + \frac{R_B}{6}(x-(S_1+a))^3 + C_1x + C_2 \quad (11) \quad \text{... .(11)}$$

Solving the above deflection equation for boundary conditions at the support points, $V_s = 0$ at $x = 0$, $V_s = 0$ at $x = (a+S_1)$ and $V_s = 0$ at $x = (a+S_1+S_2)$.

$$-\frac{Fta^3}{6} - \frac{Mta^2}{2} + C_1a + C_2 = 0 \quad \text{........... (12)}$$

$$-\frac{Ft(a+S_1)^3}{6} - \frac{Mt(a+S_1)^2}{2} + \frac{R_A}{6}{S_1}^3 + \left(C_1(a+S_1)\right) + C_2 = 0 \quad \text{..(13)}$$

$$-\frac{Ft(a+S_1+S_2)^3}{6} - \frac{Mt(a+S_1+S_2)^2}{2} + \frac{R_A}{6}(S_1+S_2)^3 + \frac{Ft(a+S_1+S_2) - R_A(S_1+S_2) + Mt)}{6S_2}{S_2}^3 + C_1(a+S_1+S_2) + C_2 = 0 \quad \text{..(14)}$$

Above equation can be written in the form matrix, AX = B, and then solved for $X = A^{-1}B$.

$$\begin{bmatrix} 0 & a & 1 \\ \frac{S_1{}^3}{6} & a+S_1 & 1 \\ \frac{(S_1+S_2)^3-(S_1+S_2)S_2{}^2}{6} & a+S_1+S_2 & 1 \end{bmatrix} X \begin{bmatrix} R_A \\ C_1 \\ C_2 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{Fta^3}{6}+\frac{Mta^2}{2} \\ \frac{Ft(a+s_1)^3}{6}+\frac{Mt(a+s_1)^2}{2} \\ \frac{Ft(a+S_1+S_2)^3}{6}+\frac{Mt(a+S_1+S_2)^2}{2}-\frac{Ft(a+S_1+S_2)S_2{}^2}{6} \\ -\frac{MtS_2{}^2}{6} \end{bmatrix}$$

Substituting and solving the above matrix gives the following values.

$R_A = 3964.54$ N

$C_1 = 1.4 \times 10^7$

$C_2 = -5.5 \times 10^8$

The deflection equation (11) can be used to find the deflection at any point after finding the unknowns by solving the above matrix function. In this case, tip of the spindle is mainly concentrated and hence the deflection at the tip of the spindle is given by:

$\delta_s = V_s(x=0) =$

Young's modulus, $E = 2 \times 10^5 N/mm^2$

Moment of inertia, $I = 3.14(D^4-d^4)/64$

$= 1.103 \times 10^6 \ mm^4$

Deflection of spindle, $\delta_s = \delta_s = \frac{-5.5 \ x \ 10^8}{2x10^5x1.103x10^6}$

$= -0.0024758$ mm

Condition 2: Rigid beam – Elastic support case.

Here in the condition values of stiffness is taken SKF catalog for all three bearings matching with dimensions.

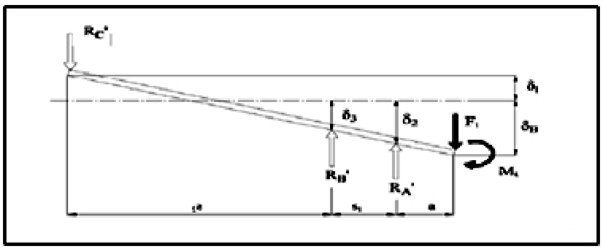

Fig 7. Rigid Beam – Elastic Support Case

From the free body diagram:

$R'_A + R'_B - R'_C = Ft$ ....(15)

$R'_A a + R'_B(a+S_1) - R'_C(a+S_1+S_2) + Mt = 0$ ....(16)

Writing the deflection at locations of R'_A, R'_B and R'_C:

$$\delta_1 = \delta_1 = -\frac{R'_C}{Kc} \tag{17}$$

$$\delta_2 = \delta_2 = -\frac{R'_A}{Kb} \tag{18}$$

$$\delta_3 = \delta_3 = -\frac{R'_B}{Kb} \tag{19}$$

Direction of deflections is opposite to the direction of forces and hence there is a negative sign in deflection. Also the beam considered here is rigid and hence deflection will be proportional.

$$\frac{\delta_1+\delta_3}{\delta_1+\delta_2} = \frac{S_1}{S_1+S_2} \quad ..... \ (20)$$

Including reaction forces in the above equation:

$$\frac{\frac{R'_B}{K_B}+\frac{R'_C}{K_C}}{\frac{R'_C}{K_C}+\frac{R'_A}{K_B}} = \frac{S_1}{S_1+S_2} \quad ....(21)$$

After solving the above equation:

$$-R'_A\frac{S_2}{K_B} + R'_B\frac{S_1+S_2}{K_B} + R'_C\frac{S_1}{K_C} = 0 \quad ....(22)$$

Writing the equilibrium and deflection equation in matrix form, GR' = H:

$$\begin{bmatrix} 1 & 1 & -1 \\ a & a+S_1 & -(a+S_1+S_2) \\ -\frac{S_2}{K_B} & \frac{S_1+S_2}{K_B} & \frac{S_1}{K_C} \end{bmatrix} X \begin{bmatrix} R'_A \\ R'_B \\ R'_C \end{bmatrix} = \begin{bmatrix} Ft \\ -Mt \\ 0 \end{bmatrix}$$

Reaction forces can be solved, $R' = G^{-1}H$: substituting the values,

$R'_A = 845$ N

$R'_B = 634$ N

$R'_C = 540$ N

Finally, deflection at the spindle tip due to spring behavior of the spindle:

$$\frac{\delta_1+\delta_3}{\delta_1+\delta_B} = \frac{S_2}{a+S_1+S_2} \quad ....(23)$$

Substituting the reaction forces and solving the equation gives:

$$\delta_B = -\frac{S_1+a}{S_2}\left(\frac{R'_B}{K_B}+\frac{R'_C}{K_C}\right) - \frac{R'_B}{K_B} \quad ... \ .... \ (24)$$

$$\delta_B = -0.001309 \text{ mm}$$

Now both δ_S, spindle deflection for the case elastic beam - rigid support case and δ_B, spindle deflection for rigid beam - elastic support case.

Total deflection of the spindle is given by:

$$\delta_{SPINDLE} = \delta_S + \delta_B$$

$$= -0.003786 \text{ mm}$$

Optimization of bearing span:

Here for the calculation of spindle deflection, bearing span S_1 of 67mm and S_2 of 360mm is considered. Now the bearing span will be varied without violating the constraints and spindle deflection will be calculated. Following this procedure enables to calculate the optimized bearing span with minimum deflection. Table no 4 illustrates the bearing span and corresponding deflection.

Table 4: Optimization of Bearing Span

Bearing span, S1(mm)	Bearing span, S2(mm)	Deflection, δ(mm)
67	360	0.003785
62	365	0.003687
57	370	0.003589
52	375	0.003491
47	380	0.003392
42	385	0.003293

From the optimization results, it's clear that there is no remarkable change in bearing deflection even by changing the bearing span. Figure no 8 shows the 3D plot of bearing span and deflection.

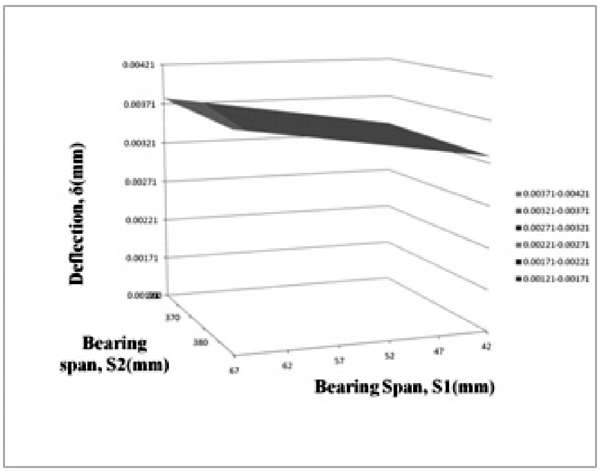

Fig 8. Deflection v/s Bearing Span

Finite Element Analysis of Spindle:

Finite element analysis has been carried out in ANSYS APDL 18.1. Deformation calculation for the combined case is calculated by providing all the parameters similar to mathematical modeling. Here Beam 188 element for Spindle and Combin 14 for bearings with the ground to body type connection is used to carry out with the analysis. For bearing consideration, damping and cross-coupling behavior are neglected in the analysis. A vertical load of 940 N is applied to the modal. Here analysis has been done for the combination case and final deflection is found during the analysis. Figure no 9 shoes the FEM results obtained from ANSYS.

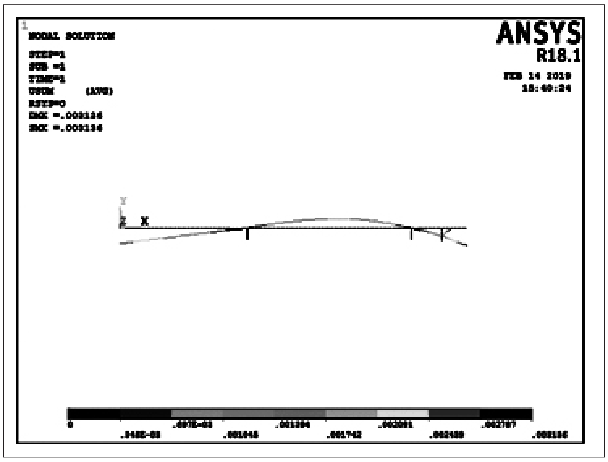

Fig 9. FEM Results of Spindle Deflection

From the finite element analysis results, deflection at the tip of the spindle is 0.003136mm. And this result is not much different from the mathematical modeling. In mathematical modeling, the spindle with uniform cross-section is considered while in the case of analysis procedure, spindle is of non uniform cross section. Even then the results won't be much varying.

RESULTS AND DISCUSSION

The spindle is an important part of the machine tool and its dynamic properties affect the overall working performance. Literature survey of the spindle system reflects that work on spindle dynamic properties is a very vast topic with never-ending improvements

Here in this paper mathematical modeling of a four support spindle of a multi spindle automat is carried out and compared with analysis results. Deflection of a spindle through analytical calculations is almost similar to ANSYS APDL results. In the analytical analysis of spindle, the spindle shaft is considered to be straight and with a uniform diameter. But in finite element analysis, a stepped shaft is included.

The maximum deflection of 0.003mm is the value for spindle deflection comparing both analytical and FEM analysis results with the given parameters. From the results it's clear that the mathematical modeling calculation used for calculating spindle deflection is correct. Optimization of bearing span is also carried out and the results prove that change in bearing span doesn't have much effect on spindle deflection. A 3D graph is also plotted to prove the above.

SUMMARY

A spindle for a multi spindle automat is designed and mathematical modeling of the spindle deflection is validated through FEM results. Optimization of the bearing span helps us to understand that the change in bearing span doesn't have much effect on the spindle deflection values.

REFERENCES

1. W R Wang, C N Chang, "Dynamic Analysis and Design of a Machine Tool Spindle-Bearing System", Transactions of the ASME, Vol .116, 281-285, 1994.
2. Jianjun Fenga, Chengzhe Lib, Zhi Wuc, "Analysis of Static and Dynamic Characteristic of Spindle System and its Structure Optimization in Camshaft Grinding Machine", Green Energy and Sustainable Development I, 020190-1- 020190-7, 2017
3. Yuzhong Cao, Yusuf Altintas, "A General Method for the Modeling of Spindle-Bearing Systems", J of Mechanical Design, Vol 126, 1089-1104, 2004
4. Xin-Yun Z., Min W., Tao Z, and Jian-Zhong H., "The Static and Dynamic Analysis of High-speed Electric Spindle Based on ANSYS", Second Int Conf. on Digital Mfg. and Automation (ICDMA) Proceedings, pp 1332-1335, 2011.
5. Book of spindle, Spindle facts, Dynomax.
6. Abele E, Altintas Y, and Brecher C, "Machine Tool Spindle Units", CIRP Annals - Mfg Technology, Vol. 59, Issue 2, pp 781-802, 2010.
7. Ayush Anand, Himel Roy, "Static and Dynamic Analysis of Lathe Spindle using ANSYS", Int J of Applied Engg. Research, ISSN 0973-4562, vol. 13, no. 9, pp. 6994-7000, (2018).
8. Mathieu Ritou, Clement Rabreau, Sebastien Le Loch, Benoit Furet, Didier Dumur, "Influence of spindle condition on the dynamic behavior", CIRP Annals-Manufacturing Technology 67, 419–422, (2018).
9. Siddesh KB, Dr. B N Ravikumar, Ramesh H Araluguppi, "Dynamic analysis of spindle shaft", Int J for Research in Applied Science & Engineering Technology, May 2016.
10. Zhu L., Zhu C., Yu T., Shi J., and Wang W., "Dynamic Analysis and Design of the Spindle-Bearing System in Turn-milling Centre", IEEE International Conf. on Automation and Logistics Procd, pp 2394-2398, 2007.
11. Altintas Y. and Budak E., "Analytical Prediction of Stability Lobes in Milling", CIRP Annals - Manufacturing Technology, Vol. 44, Issue 1, pp 357-362, 1995.
12. Altintas Y. and Cao Y., "Virtual Design and Optimization of Machine Tool Spindles", CIRP Annals - Mfg Technology, Vol. 54, Issue 1, pp 379-382, 2005.
13. Zhu L., Zhu C., Yu T., Shi J., and Wang W., "Dynamic Analysis and Design of the Spindle-Bearing System in Turn-milling Centre", IEEE Int Conf. on Automation and Logistics Proceedings, pp 2394-2398, 2007.
14. Sandvik, Training hand book, Metal cutting technology, 2017.
15. Sandvik, Turning tools, 1-284, 2017.
16. SKF, General Catalogue, 2003.
17. SKF, Rolling bearings, 2018

G S Pramod Krishna is presently working as Scientist - B, DDC Department, Central Manufacturing Technology Institute, Bangalore, Karnataka. He has obtained both his B Tech degree in 2012 and M.Tech(Manufacturing) in 2017 from NIT Warangal, Telangana. He has 4 years of experience in Mechanical System Design.

Tom Saju is presently doing his M Tech degree in Advanced Manufacturing Technology from Karunya Institute of Technology and Science, Coimbatore. He has completed his B Tech degree from Amal Jyothi College of Engineering, Kanjirapally. He is currently doing his M.Tech Project titled 'Dynamic and Thermal Analysis of spindles for Multi Spindle Automat' in CMTI, Bangalore. (E-mail; tomvelassery@gmail.com)