Modal analysis of uniform short cantilever beam using theoretical and experimental approaches*

Yemineni Siva Sankara Rao*[1] and Kutchibotla Mallikarjuna Rao[2]

*[12]Mechanical Engineering Department, University College of Engineering, JNTUK, Kakinada, India

*Presented in 1st National Conference on Smart Manufacturing & Industry 4.0 (NCSMI4) at Central Manufacturing Technology Institute (CMTI), Bengaluru, during May 30-31, 2019.

Keywords:
Short Uniform Rectangular Cantilever Beam,
Mild steel,
Experimental Modal Analysis using FRF,
Timoshenko Beam Theory,
Euler-Bernoulli Beam Theory,
ANSYS R14.5

ABSTRACT

In this investigation, experimental, theoretical models like Timoshenko Beam Theory (TBT), Euler-Bernoulli Beam Theory (EBT), and simulation software ANSYS R14.5 are used for executing the free transverse vibration analysis of a short cantilever beam for estimating the fundamental natural frequencies for different lengths of uniform short cantilever beam. The cross-section of the cantilever beam is rectangular and the material used for analysis is Mild Steel. The obtained results from experimental, both analytical theories, and simulation software ANSYS R14.5 are compared. Although the results obtained from both analytical models are closer to each other, results from Timoshenko beam model are nearer to those of results from ANSYS R14.5 and then to the experimental results than those obtained from the Euler-Bernoulli beam model. Also, for an arbitrary length of cantilever beam natural frequencies for the first four modes are obtained from ANSYS R14.5, TBT, and EBT models and are compared. Here also TBT model, ANSYS R14.5 results are nearer to each other than the results from EBT model. Hence, Timoshenko beam model is accurate theoretical model in analyzing the short cantilever beams for their modal analysis.

1. Introduction

Structures behave dynamically when they are subjected to loads or displacements. The most popular theories for the dynamic analysis of beams are Euler-Bernoulli beam theory and Timoshenko beam theory. When the beams are subjected to loads the primary effects of are transverse deflection due to pure bending and transverse inertia and the secondary effects are shear deformation and rotatory inertia of the cross-section of the beam. The governing equation which includes the primary effects only is Euler-Bernoulli beam and that which include secondary effects along with primary effects is Timoshenko beam [3]. The Euler-Bernoulli beam theory can give accurate results for modal analysis for lower modes of long cantilever beams (L/D>7) quite accurately. For higher modes of long cantilever beams and for short cantilever beams (L/D<7) Timoshenko beam theory will give the accurate values for their modal analysis. In the year 1921, the effects of shear deformation and rotatory inertia were first introduced in the vibrating beam equations by Timoshenko, S. P. Timoshenko, in that article, considered a value of 2/3 as shear coefficient, *K* for rectangular cross section [1].

The shear coefficient, *K* is introduced for allowing the fact that the shear stress is non uniform across the cross section. According to the definition, the shear coefficient, *K*is the ratio of the average shear strain on a section to the shear strain at the centroid. It is a dimensionless quantity, dependent on the shape of the cross section, and is considered because the shear stress and shear strain are not uniformly distributed over the cross section. The one-dimensional theory of beams can be improved by considering the transverse shear deformations and, in the case of vibrating beams, rotary inertia. The beam equations which consider these effects are generally called as Timoshenko's beam equations [1, 2]

*Corresponding author,
E-mail: siva.cadcam@gmail.com (Yemineni Siva Sankara Rao)

and have been received considerable attention in the literature. Timoshenko uses the values $K = (6+12\vartheta+6\vartheta^2) / (7+12\vartheta+4\vartheta^2)$ for the circular cross section and $K = (5+5\vartheta)/(6+5\vartheta)$ for the rectangular cross section and are nearer to the experimental values [2].

Cowper, G. R., 1966, derived the shear coefficients formulae for various cross sections like circular, hollow circular, rectangular, elliptical, semi-circular, and thin-walled round tubular, square tubular, I-Section, Box section, Spar-And-Web, T-Section cross sections for static problems while deriving the equations of Timoshenko's beam theory by integration of the equations of three-dimensional elasticity theory. The numerical results obtained from above formulae are agreed with Timoshenko's results when Poisson's ratio value, ϑ is taken as zero [4].

Kaneko concluded that the values obtained for K obtained from Timoshenko's [2] equations are closer to the experimental values [5].

Hutchinson and Zilmer compared their three dimensional series solution and a plane stress solution for the completely free beam with the Timoshenko beam theory for rectangular cross section. The plane stress solution is in good agreement with Timoshenko beam theory using Timoshenko's shear coefficient, K [6].

However, in this article, the value obtained from the equation, $K = (5+5\vartheta)/(6+5\vartheta)$ [2], suggested by Timoshenko, S. P. for shear coefficient for rectangular cross section is used.

2. Governing Partial Differential Equations for Free Transverse Vibrations of a Beam

In this investigation, for free transverse vibrational analysis a cantilever beam is considered (Ref. Fig .1).

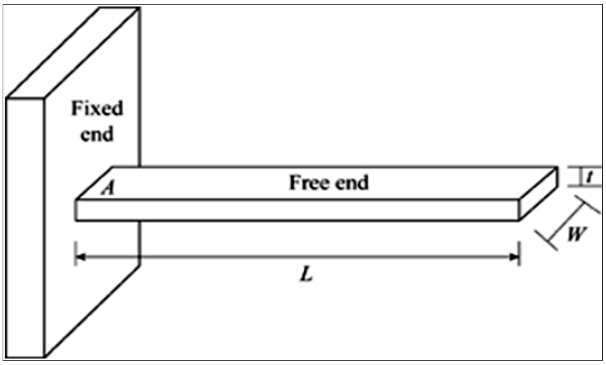


Fig. 1. Cantilever beam.

2.1. *Governing partial differential equations for free transverse vibrations of a uniform timoshenko beam [8]*

In Timoshenko beam theory the effects of shear deformation (SD) and rotary inertia (RI) are considered for the flexural vibrations of a uniform rectangular short beam. The loading condition and free body diagram of a cantilever beam according to Timoshenko Beam Theory is shown in the Fig 2.

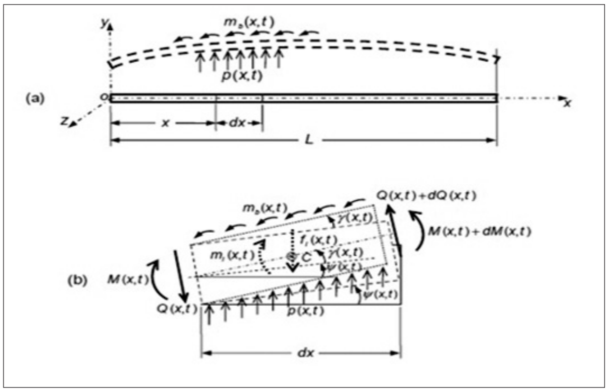


Fig. 2. A transversely vibrating timoshenko beam [8]. a) External loads, p(x) and m_b(x, t), in the coordinate system oxyz; b) free - body diagram for the beam segment dx.

For free transverse vibration analysis of a beam the external force per unit length, ***p(x, t)*** and the external bending moment per unit length, $m_b(x, t)$ can be neglected. Now the coupled equations for the total deflection, $y(x, t)$ and the rotation due to bending moment, $\psi(x, t)$ using Timoshenko beam theory are given by

$$\rho A \frac{\partial^2 y(x,t)}{\partial t^2} - KGA\left[\frac{\partial^2 y(x,t)}{\partial x^2} - \frac{\partial \psi(x,t)}{\partial x}\right] = 0 \quad (1a)$$

$$EIz \frac{\partial^2 \psi(x,t)}{\partial x^2} + KGA\left[\frac{\partial y(x,t)}{\partial x} - \psi(x,t)\right] - \rho Iz \frac{\partial^2 \psi(x,t)}{\partial t^2} = 0 \quad (1b)$$

Where, E, Iz, ρ, A, K, G are modulus of elasticity, second moment of area, mass density, cross-sectional area, shear coefficient and shear modulus of the beam respectively. And also here K, the shear coefficient.

Eliminating $\psi(x, t)$ or $y(x, t)$ from the equations (1a) and (1b) we respectively get two differential equations in $y(x, t)$ and $\psi(x, t)$ as follows:

$$EIz \frac{\partial^4 y(x,t)}{\partial x^4} + \rho A \frac{\partial^2 y(x,t)}{\partial t^2} - \rho Iz\left(1 + \frac{E}{KG}\right)\frac{\partial^4 y(x,t)}{\partial x^2 \partial t^2} + \frac{\rho^2 Iz \partial^4 y(x,t)}{KG \partial t^4} = 0 \quad (2a)$$

$$EIz \frac{\partial^4 \psi(x,t)}{\partial x^4} + \rho A \frac{\partial^2 \psi(x,t)}{\partial t^2} - \rho Iz\left(1 + \frac{E}{KG}\right)\frac{\partial^4 \psi(x,t)}{\partial x^2 \partial t^2} + \frac{\rho^2 Iz \partial^4 \psi(x,t)}{KG \partial t^4} = 0 \quad (2b)$$

The total slope of the Timoshenko beam at an arbitrary time is given by the following equation:

$$\frac{\partial y(x,t)}{\partial x} = \psi(x,t) + \gamma(x,t) \quad (3a)$$

Where,

$\psi(x, t)$ is the rotational angle due to the bending moment and $y(x, t)$ is the shear strain due to the shearing force. From the above equation

$$\gamma(x,t) = \frac{\partial y(x,t)}{\partial x} - \psi(x,t) \quad (3b)$$

For the case of free vibrations, the translational and rotational displacement functions respectively will be assumed as follows:

$$y(x,t) = Y(x)e^{i\omega t} \quad (4a)$$

$$\psi(x,t) = \boldsymbol{\psi}(x)e^{i\omega t} \quad (4b)$$

Where,

$Y(x)$ and $\boldsymbol{\psi}(x)$ are the amplitudes of $y(x, t)$ and $\psi(x, t)$ respectively, and is the circular frequency in rad/sec, t is the time in seconds and $i=\sqrt{-1}$.

Now, the solutions for the equations 2(a) and 2(b) from the equations (4a) and (4b) and eliminating the common term e^{iwt} will be as follows:

$$Y(x) = \bar{A}cosh\delta x + \bar{B}sinh\delta x + \bar{C}cos\varepsilon x + \bar{D}sin\varepsilon x \quad (5a)$$

Where, the constants $\bar{A}, \bar{B}, \bar{C}, and\ \bar{D}$ are for the translational displacement function $Y(x)$, and

$$\boldsymbol{\psi}(x) = A'sin\delta x + B'cos\delta x + C'sin\varepsilon x + D'cos\varepsilon x \quad (5b)$$

Where, the constants A', B', C', and D' are for the rotational displacement function $\boldsymbol{\psi}(x)$. The equations $Y(x)$, $\boldsymbol{\psi}(x)$ represent the translational and rotational mode shapes of the uniform Timoshenko beam respectively. In equations (5a, b),

$$\delta = \sqrt{\frac{1}{2}\left(-\alpha + \sqrt{\alpha^2 + 4\beta^4}\right)}$$

$$\varepsilon = \sqrt{\frac{1}{2}\left(\alpha + \sqrt{\alpha^2 + 4\beta^4}\right)} \quad (6a, b)$$

Where,

$$\alpha = \frac{\rho Iz\left(1 + \frac{E}{KG}\right)\omega^2}{EIz}$$

$$\beta^4 = \frac{\rho A\omega^2 - \frac{\rho^2 Iz}{KG}\omega^4}{EIz} \quad (7a, b)$$

$A'=a\bar{B}$, $B'=a\bar{A}$, $C'=-b\bar{D}$ and $D'=-b\bar{C}$ (8a, b)

Where,

$$a = \frac{KGA\delta}{(-EIz\delta^2 - \rho Iz\omega^2 + KGA)},$$

$$b = \frac{KGA\varepsilon}{(-EIz\varepsilon^2 - \rho Iz\omega^2 + KGA)} \quad (9a, b)$$

Also, on substituting equations (4a, b) and $\gamma(x, t) = \Gamma(x)e^{iwt}$ into equations (3b) and then inserting equations (5a, b) into the resulting expression, one obtains the shear deformations displacement function as follows:

$$\Gamma(x) = Y'(x) - \psi(x) = cAsinh\delta x + cBcosh\delta x - dCsin\varepsilon x + dDcos\varepsilon x \quad (10)$$

Where,

$c = \delta - a$, $d = \varepsilon - b$ (11a, b)

We can determine the integrating constants $\bar{A}, \bar{B}, \bar{C}, and\ \bar{D}$ by using boundary conditions of the beam.

For free vibrations, we have

$$Q(x,t) = Q(x)e^{i\omega t},$$

$$M(x,t) = M(x)e^{i\omega t}, and$$

$$\gamma(x,t) = \Gamma(x)e^{i\omega t} \quad (12a, b, c)$$

Where,

$Q(x)$, $M(x)$, and $\Gamma(x)$ represent the amplitudes of shear force, $Q(x, t)$, bending moment, $M(x, t)$, and shear strain, (x, t) respectively. And also we have

$$Q(x) = KGA\,\Gamma(x) \quad (13a)$$

$$M(x) = EIz\,\boldsymbol{\psi}'(x) \quad (13b)$$

2.1.1. Boundary conditions for calculating the constants $\bar{\mathbf{A}}, \bar{\mathbf{B}}, \bar{\mathbf{C}}, \mathbf{and}\ \bar{\mathbf{D}}$

For Free End:

$$M(x) = EIz\,\boldsymbol{\psi}'(x) = 0 \text{ and } Q(x) = KGA\,\Gamma(x) = 0 \quad (14a, b)$$

For Clamped End:

$$Y(x) = 0 \text{ and } \boldsymbol{\psi}(x) = 0 \quad (15a, b)$$

For Hinged End:

$$Y(x) = 0 \text{ and } M(x) = EIz\,\boldsymbol{\psi}'(x) = 0 \quad (16a, b)$$

From the equations (15a, b) and (14a, b) the boundary conditions for C-F beam are respectively as follows:

$$Y(0) = 0, \boldsymbol{\psi}(0) = 0 \quad (17a, b)$$

$$\boldsymbol{\psi}'(L) = 0, \Gamma(L) = 0 \quad (18a, b)$$

From the equations (5a), (5b) and (10), and (17a, b), (18a, b), we have

$$Y(0) = \bar{A} + \bar{C} = 0 \quad (19a)$$

$\psi(0) = a\bar{B} + b\bar{D} = 0$ (19*b*)

$\psi'(L) = a\delta\bar{A}cosh\ \delta L + a\delta\bar{B}sinh\delta L - b\varepsilon\bar{C}cos\varepsilon L - b\varepsilon\bar{D}sin\ \varepsilon L = 0$ (19c)

$\Gamma\ (L) = c\bar{A}sinh\ \delta L + c\bar{B}\ cosh\ \delta L - d\bar{C}\ sin\ \varepsilon L + d\bar{D}cos\ \varepsilon L = 0$ (19d)

For non-trivial solution of equations (19a)-(19d) requires that

$$\Delta(\omega) = \begin{vmatrix} 1 & 0 & 1 & 0 \\ 0 & a & 0 & b \\ a\delta cosh\ \delta L & a\delta\ sinh\ \delta L & -b\varepsilon\ cos\varepsilon L & -b\varepsilon\ sin\ \varepsilon L \\ c\ sinh\ \delta L & c\ cosh\ \delta L & -d\ sin\ \varepsilon L & d\ cos\ \varepsilon L \end{vmatrix} = 0 \quad (20)$$

and is the frequency equation of the C-F uniform Timoshenko beam which is a transcend equation.

Now, from equations (6a, b), (7a, b), (9a, b), and (11a, b), the equation (20) is a function of ω, and the any one of the numerical methods (such as half-interval method) can be used to calculate the natural frequencies ω_r (r = 1, 2, 3 … represents mode number) [10].

2.2. Governing partial differential equations for free transverse vibration of a uniform euler-bernoulli beam [8]

For the free transverse vibration of slender beam EBT neglect the external force per unit length *p(x, t)*, and external bending moment per unit length, $m_b(x, t)$.

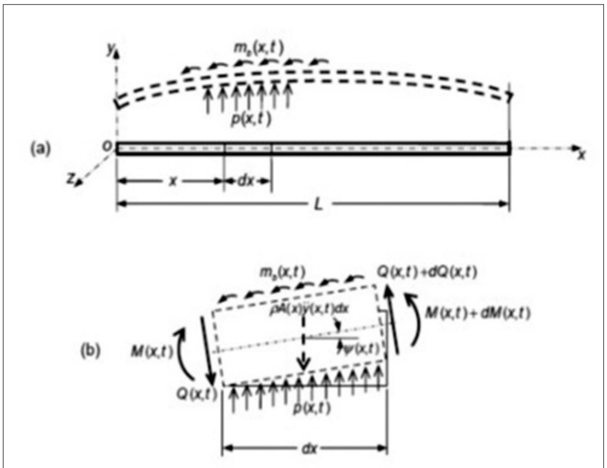


Fig 3. A Transversely vibrating euler-bernoulli beam [8].

a) The external force, p(x) and the external moment, m_b(x, t), in the coordinate system oxyz;

b) The free-body diagram for the differential beam segment dx.

In Euler-Bernoulli beam theory we neglect the shear deformation (SD) and rotary inertia (RI), hence the differential equations for free transverse vibration are:

$$Q(x,t) = -EIz\frac{\partial^3 y(x,t)}{\partial x^3} \quad (21)$$

$$EIz\frac{\partial^4 y(x,t)}{\partial x^4} + \rho A\frac{\partial^2 y(x,t)}{\partial t^2} = 0 \quad (22)$$

Where, *Q(x, t)* is the shear force and *E, Iz, and A* are the young's modulus, second moment of the area of the beam cross section, mass density, and cross-sectional area of the uniform beam respectively. Consider the displacement function as:

$$y(x,t) = Y\ (x)e^{i\omega t} \quad (23)$$

Where, *Y*(*x*) is the amplitude of *y(x, t), ω* is the circular frequency of the beam, t is the time, and $i=\sqrt{-1}$.

From equations (22) and (23), we will get

$$Y''\,''(x) - \beta^4 Y(x) = 0 \quad (24)$$

or

$$\omega = (\beta L)^2\sqrt{\frac{EI_Z}{(\rho A L^4)}} \quad (25)$$

Now, the solution of the equation (24) will be in the form:

$$Y(x) = B_1 e^{\beta x} + B_2 e^{-\beta x} + B_3 e^{i\beta x} + B_4 e^{-i\beta x} \quad (26)$$

Where,

$B_{1,}\ B_{2,}\ B_3$ and B_4 are constants of integration.

And since,

$$e^{\pm\beta x} = cosh\beta x \pm sinh\beta x, e^{\pm i\beta x} = cosh\beta x \pm i\ sinh\beta \quad (27)$$

Equation (26) can also be expressed as:

$$Y(x) = C_1 cosh\beta x + C_2 sinh\beta x + C_3 cos\beta x + C_4 sin\beta x \quad (28)$$

and is the natural mode shape of the uniform Euler-Bernoulli beam with the integration constants C_1-C_4 which can be determined from boundary conditions of the beam.

2.2.1. Boundary conditions for calculating integrating constants $C_{1,}\ C_{2,}\ C_{3,}\ C_4$

Once again recalling

$$\psi(x,t) = \frac{\partial y\,(x,t)}{\partial x} \quad (29)$$

is the bending slope,

$$\psi\ (x) = Y'(x) \quad (30)$$

is the amplitude of $\psi(x,t)$,

$$\bar{M}(x,t) = EIzY(\acute{x}) \quad (31)$$

is the amplitude of bending moment, and

$$\bar{Q}(x) = -EIzY'''(x) \quad (32)$$

is the amplitude of *Q(x, t),*

Now, boundary conditions for determining the integrating constants C_1-C_4 are similar to as expressed in equations (14a, b), 15(a, b), and (16a, b) for free, clamped, and hinged ends respectively and are as follows:

At the clamped end (*x*=0):

$$Y(0) = 0,\ Y'(0) = 0 \quad (33a,b)$$

At the free end (*x* = L):

$$Y''(L) = 0,\ Y''''(L) = 0 \quad (34a,b)$$

Now, from the equations (28), (33a, b) we will get

$$C3 = -C1, C4 = -C2 \quad (35a,b)$$

On substituting the equations (35a, b) into equation (28) we will get

$$Y = C1\,(cosh\beta x - cos\beta x) + C2(sinh\beta x - sin\beta x) \quad (36)$$

Now from the equations (36) and (34a, b), we will obtain

$$C1\,(cosh\beta L + cos\beta L) + C2(sinh\beta L + sin\beta L) = 0 \quad (37a)$$

$$C1\,(sinh\beta L - sin\beta L) + C2\,(cosh\beta L + cos\beta L) = 0 \quad (37b)$$

For the non-trivial solution of the simultaneous equations (37a, b), it requires that

$$\Delta(\omega) = \begin{vmatrix} \cosh\beta L + \cos\beta L & \sinh\beta L + \sin\beta L \\ sinh\beta L - sin\beta L & cosh\beta L + cos\beta L \end{vmatrix} = 0 \quad (38)$$

or

$$\cosh\beta L \cos\beta L = -1 \quad (39)$$

or

$$\cos\beta L = -\frac{1}{\cosh \beta L} \quad (40)$$

The above equation (40) is called the frequency of the Euler-Bernoulli beam and for the solution of this equation (40) we can use a numerical method like Half- Interval method [10].

3. Results and Discusions

For validating the fundamental natural frequencies obtained from ANSYS R14.5, EBT and TBT with experimental values we considered a numerical example of a cantilever beam with different lengths as 137.5mm, 112.5mm, 87.5mm, and 62.5mm and the cross-sectional dimensions as 24.75mm width and 10 mm thickness. The material used is mild steel. The element selected for the beam definition is BEAM 2D 188 in ANSYS R14.5 [9].

Table 1.
Mechanical properties of mild steel.

S. No.	Mechanical Property	Value
1	Young's Modulus	1.96×10^{11} Pa
2	Density	7850 Kg/m^3
3	Poison's Ratio	0.3

3.1. Ansys R14.5 Results [9]

The following figures named as Fig. 4, 5, 6, and 7 are the fundamental mode shapes and the corresponding fundamental natural frequencies of mild steel cantilever beams of various effective lengths 137.5 mm, 112.5mm, 87.5mm, and 62.5 mm respectively obtained in ANSYS R14.5.

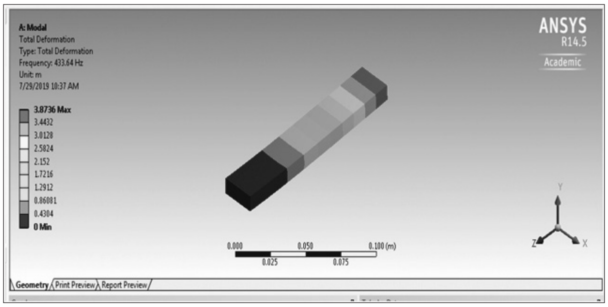


Fig 4. For Length, L=137.5 mm

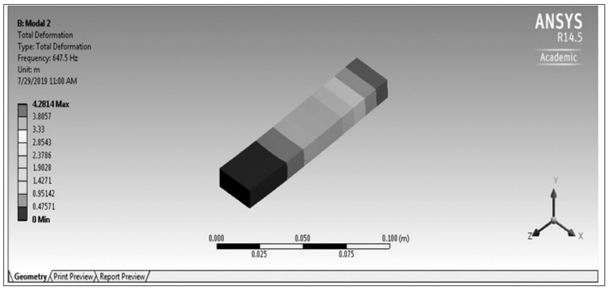


Fig 5. For Length, L=112.5 mm

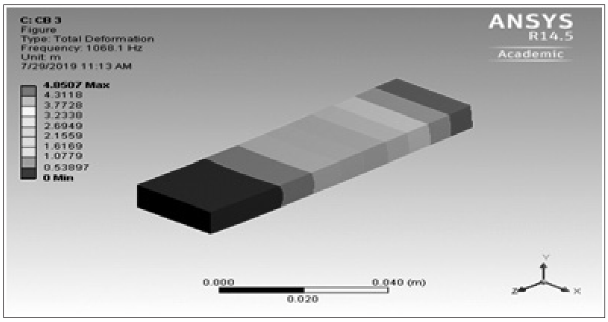


Fig 6. For Length, L=87.5 mm

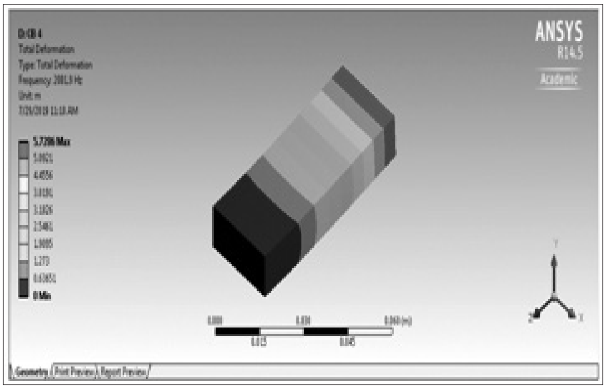


Fig 7. For length, L=62.5mm

From Table 2, we can observe that the fundamental natural frequency values of a cantilever beam increases as the length of the cantilever beam decreases. We can also observe that the fundamental natural frequency values obtained for identical cantilever beams in length and cross-section (rectangular) from Timoshenko Beam Theory (TBT) are closer to the experimental values than those than those obtained from ANSYS R14.5, and Euler-Bernoulli Theory (EBT). The percentage variation of the fundamental natural frequency value for the decrease in length of the cantilever beam for their identical cross-section (rectangular) obtained from the ANSYS R14.5 is increased more than that from Timoshenko Beam Theory (TBT), Euler-Bernoulli Theory (EBT) when compared with the values obtained from experimentation (Ref. Graph-1).

The figures named as Fig 8, 9, 10, 11 respectively show the first four principal mode shapes and the corresponding natural frequencies of the mild steel cantilever beam of length, L = 137.5 mm in ANSYS R14.5.

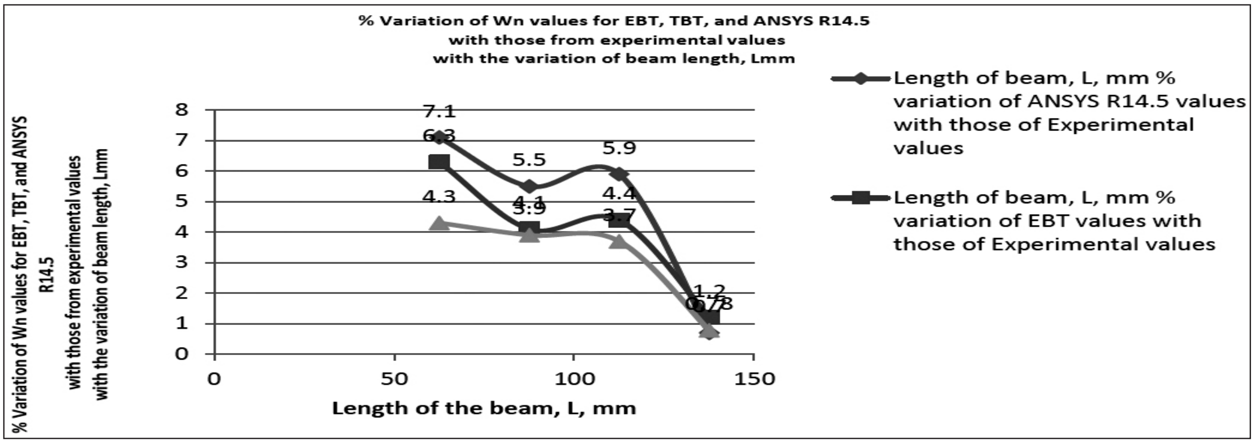
From Table 3, we can observe that the percentage variation of EBT values with ANSYS R14.5 values for the natural frequency of the first four modes for the rectangular cantilever beam of length 137.5 mm is from -1.54% to 9.26%. And also the percentage variation of TBT values with ANSYS R14.5 values for the natural frequencies of the first four modes of a cantilever beam of length 137.5 mm and identical rectangular cross-section is from -1.95% to -2.09% for the shear coefficient value, K=0.8667. The values obtained for TBT are nearer to the ANSYSR14.5 values than those for EBT values for short cantilever beam. The same can be observed in the following Graph 2.

Table 2.
Comparison of ANSYS R14.5 results, the results of EBT and TBT for different lengths of mild steel cantilever beam with experimental results.

S. No.	Length of beam, L, mm	Experimental Results (FRF Analysis)	ANSYS R14.5 Results	EBT Results	TBT Results K = 0.8667	% variation of ANSYS R14.5 values with those of Experimental values	% variation of EBT values with those of Experimental values	% variation of TBT values for K = 0.8667 with those of Experimental values
		ω_n, Hz	ω_n, Hz	ω_n, Hz	ω_n, Hz			
1	137.5	421.875	433.64	426.95	425.18	0.7	1.2	0.78
2	112.5	611.15	647.5	637.78	633.9	5.9	4.4	3.7
3	87.5	1012.15	1068.1	1053.85	1052.13	5.5	4.1	3.9
4	62.5	1943.75	2081.9	2066.4	2026.4	7.1	6.3	4.3

Table 3.
Comparison of ANSYS R14.5 Results with the results of EBT and TBT for mild steel cantilever beam of length, L = 137.5 mm for first four modes.

Mode No.	ANSYS R14.5 ω_n, Hz	EBT ω_n, Hz	TBT ω_n, Hz K = 0.8667	% variation of EBT values with ANSYS R14.5	% variation of TBT values for K = 0.8667 with ANSYS R14.5
1	433.64	426.95	425.18	-1.54	-1.95
2	2653.4	2675.55	2602.5	0.83	-1.91
3	7176.4	7.49E + 03	7034	4.36	-1.98
4	13437	14681	13,156.70	9.26	-2.09



Graph 1. Percentage variation of fundamental natural frequency values for different lengths of cantilever beam using ANSYS R14.5, EBT and TBT with those obtained in Experimentation.

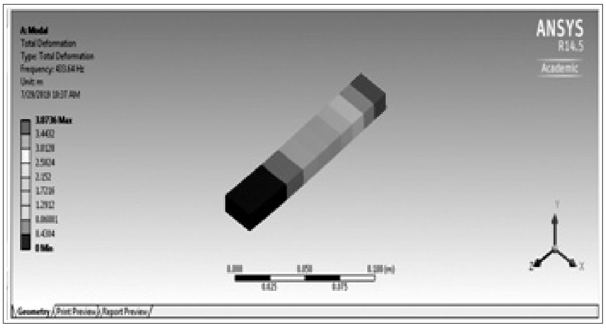


Fig 8. Principal Mode 1.

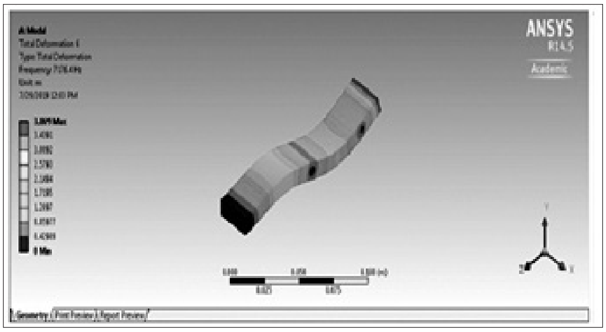


Fig 10. Principal mode 3.

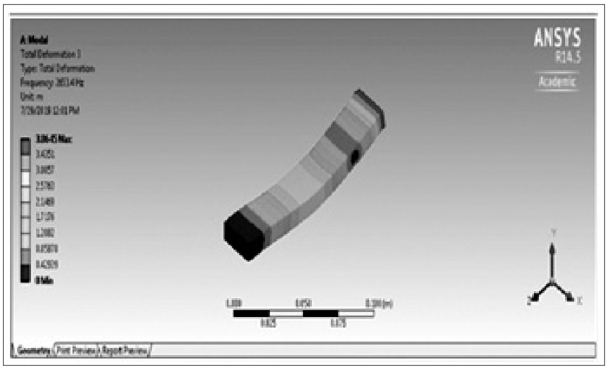


Fig 9. Principal mode 2.

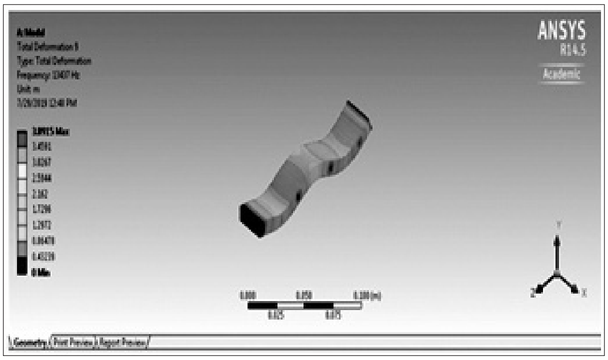
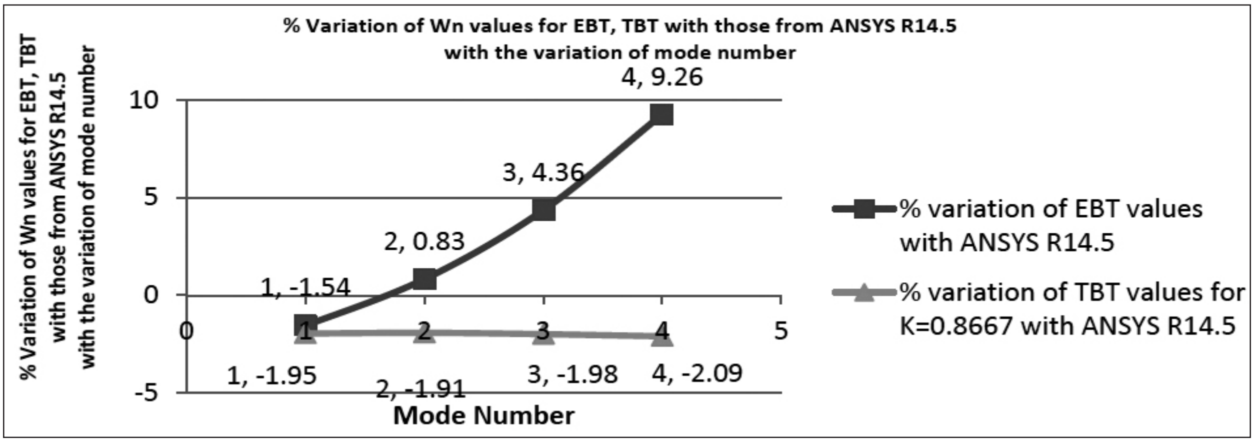


Fig 11. Principal mode 4.



Graph 2. Percentage variation of EBT, TBT frequency ω_n,Hz values with those from ANSYS R14.5 for the first four Modes.

Conclusions

1. We can observe that the fundamental natural frequency values of a cantilever beam will increase as the length of the cantilever beam reduces keeping the cross-section constant. And also we can observe that the fundamental natural frequency values for different lengths of the identical rectangular cross-section cantilever beam from Timoshenko Beam Theory (TBT), Euler-Bernoulli Theory (EBT) are nearer to the experimentation values than those obtained from ANSYS R14.5.
2. The percentage variation of the fundamental natural frequencies for the decrease of the length of the cantilever beam obtained from the Euler-Bernoulli Theory (EBT) are more than those from the Timoshenko Beam Theory (TBT), ANSYS R14.5 when compared with the values obtained from experimentation.
3. We can also observe that the variation of EBT and TBT, ANSYS R14.5 values of fundamental natural frequency with those of experimentation for a fixed length of cantilever beam and identical cross-section is increased as the mode number is increased.
4. The fundamental natural frequencies obtained from Timoshenko Beam Theory (TBT), are nearer to the ANSYS R14.5 than those values obtained from Euler-Bernoulli Theory (EBT).

Hence, we can say that for the analysis of free transverse vibrations of short cantilever beams either for lower or higher modes Timoshenko Beam Theory (TBT) is the best estimate than Euler-Bernoulli Beam Theory (EBT).

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Y Siva Sankara Rao, working as Assistant Prof. in Mechanical Engineering Dept., 'Vignan's LARA Institute of Technology & Science, Guntur, A. P'. He graduated in Mechanical Engineering at 'Nimra College of Engineering and Technology, Vijayawada, A. P'. and awarded M. Tech in the stream of CAD/CAM at 'UCE, JNTUK, Kakinada, A. P.' He has 6 years of teaching experience. He is presently pursuing PhD in Solid Mechanics in UCE, JNTUK, Kakinada, A. P. He had published 5 technical papers in international journals and conferences, guided to fabricate four working models and simulation projects for B. Tech students.

Dr. K Mallikarjuna Rao, retired prof. and presently working as Adjunct faculty in Mechanical Engineering department in 'UCE, JNTUK, Kakinada, A. P.' He completed B. Tech in Mechanical Engineering at REC Warangal, A. P., India, M. E. in the stream of Machine Design at IISc, Bangalore, Karnataka, India, M. Engg., in the stream of Industrial Engineering and Management at AIT, Bangkok, Thailand. He pursued PhD in Finite Elements, at IISc Bangalore, Karnataka, India. His Achievements includes recipient of State Govt. Best teacher award in 2011, Bagged the Visweswaraya Engineer of the year award for 2010, Best Teacher UCEK, JNTUK for the year 2011. His research activities include 10 PhD's guided, 10 PhD's submitted, 5 PhD's being guided presently and about 100 M. Tech theses guided. His Publications include 40 international journals, 5 international conferences, 2 national journals, 16 national conferences.